POWERING SIMPLE BEAM ELEMENTS WITH DETAILED 3D FEA FIDELITY

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The Problem

Length: 8.6 m
Chord: 0.72 m.
D-spar: 60 graphite/epoxy plies
Ply thickness: 125 microns

3D FEA
At least one solid elements/layer
10^9 DOFs/blade
Not suitable for design & optimization
Smeared property approach improves efficiency but loses significant accuracy
VABS: Beam Constitutive Modeling

3D Elasticity

\[
\sigma_{ij,j} + f_i = 0
\]

\[
\varepsilon_{ij} = \frac{1}{2} (w_{i,j} + u_{j,i})
\]

\[
\begin{pmatrix}
\sigma_{11} \\
\sigma_{22} \\
\sigma_{33} \\
\sigma_{23} \\
\sigma_{13} \\
\sigma_{12}
\end{pmatrix}
= 
\begin{pmatrix}
c_{11} & c_{12} & c_{13} & c_{14} & c_{15} & c_{16} \\
c_{12} & c_{22} & c_{23} & c_{24} & c_{25} & c_{26} \\
c_{13} & c_{23} & c_{33} & c_{34} & c_{35} & c_{36} \\
c_{14} & c_{24} & c_{34} & c_{44} & c_{45} & c_{46} \\
c_{15} & c_{25} & c_{35} & c_{45} & c_{55} & c_{56} \\
c_{16} & c_{26} & c_{36} & c_{46} & c_{56} & c_{66}
\end{pmatrix}
\begin{pmatrix}
\varepsilon_{11} \\
\varepsilon_{22} \\
\varepsilon_{33} \\
2\varepsilon_{23} \\
2\varepsilon_{13} \\
2\varepsilon_{12}
\end{pmatrix}
\]

Reference line

1D beam analysis

\[
\begin{align*}
\gamma_{11} &= \bar{u}_1' \\
\kappa_1 &= \Phi_1' \\
\kappa_2 &= -\bar{u}_3'' \\
\kappa_3 &= \bar{u}_2''
\end{align*}
\]

\[
\begin{align*}
\frac{dF_1}{dx_1} + p_1 &= 0 \\
\frac{dM_1}{dx_1} + q_1 &= 0 \\
\frac{d^2 M_2}{dx_1^2} + p_3 + \frac{dq_2}{dx_1} &= 0 \\
\frac{d^2 M_3}{dx_1^2} - p_2 + \frac{dq_3}{dx_1} &= 0
\end{align*}
\]
Introduction to VABS Theory

- Minimize kinetic energy loss.
- A diagonal mass matrix is possible if blade axis is at the mass center and sectional coordinates are the principal inertia axes.

\[
\mathbf{K} = \frac{1}{2} \begin{pmatrix} V_1 \\ V_2 \\ V_3 \\ \Omega_1 \\ \Omega_2 \\ \Omega_3 \end{pmatrix}^T \begin{bmatrix} \mu & 0 & 0 & 0 & \mu x_{m3} & -\mu x_{m2} \\ \mu & 0 & -\mu x_{m3} & 0 & 0 & 0 \\ \mu & \mu x_{m2} & 0 & 0 & 0 & 0 \\ i_{22} + i_{33} & 0 & 0 & 0 & 0 & 0 \\ \text{symmetric} & i_{22} & -i_{23} & 0 & 0 & 0 \\ i_{33} & i_{33} & 0 & 0 & 0 & 0 \end{bmatrix} \begin{pmatrix} V_1 \\ V_2 \\ V_3 \\ \Omega_1 \\ \Omega_2 \\ \Omega_3 \end{pmatrix}
\]
Introduction to VABS Theory

- Euler-Bernoulli model

\[
\begin{align*}
\begin{bmatrix} F_1 \\ M_1 \\ M_2 \\ M_3 \end{bmatrix} &= \begin{bmatrix} S_{11} & S_{12} & S_{13} & S_{14} \\ S_{12} & S_{22} & S_{23} & S_{24} \\ S_{13} & S_{23} & S_{33} & S_{34} \\ S_{14} & S_{24} & S_{34} & S_{44} \end{bmatrix} \begin{bmatrix} \gamma_{11} \\ \kappa_1 \\ \kappa_2 \\ \kappa_3 \end{bmatrix}
\end{align*}
\]

- Timoshenko model

\[
\begin{align*}
\begin{bmatrix} F_1 \\ F_2 \\ F_3 \\ M_1 \\ M_2 \\ M_3 \end{bmatrix} &= \begin{bmatrix} S_{11} & S_{12} & S_{13} & S_{14} & S_{15} & S_{16} \\ S_{12} & S_{22} & S_{23} & S_{24} & S_{25} & S_{26} \\ S_{13} & S_{23} & S_{33} & S_{34} & S_{35} & S_{36} \\ S_{14} & S_{24} & S_{34} & S_{44} & S_{45} & S_{46} \\ S_{15} & S_{25} & S_{35} & S_{45} & S_{55} & S_{56} \\ S_{16} & S_{26} & S_{36} & S_{46} & S_{56} & S_{66} \end{bmatrix} \begin{bmatrix} \gamma_{11} \\ 2\gamma_{12} \\ 2\gamma_{13} \\ \kappa_1 \\ \kappa_2 \\ \kappa_3 \end{bmatrix}
\end{align*}
\]

- 1D beam analysis should be changed to allow fully populated stiffness matrices.
Introduction to VABS Theory

- Vlasov model: important for thin-walled beams with open sections

\[
\begin{bmatrix}
F_1 \\
M_1 \\
M_2 \\
M_3 \\
M_\omega
\end{bmatrix}
= 
\begin{bmatrix}
S_{11} & S_{12} & S_{13} & S_{14} & S_{15} \\
S_{12} & S_{22} & S_{23} & S_{24} & S_{25} \\
S_{13} & S_{23} & S_{33} & S_{34} & S_{35} \\
S_{14} & S_{24} & S_{34} & S_{44} & S_{45} \\
S_{15} & S_{25} & S_{35} & S_{45} & S_{55}
\end{bmatrix}
\begin{bmatrix}
\gamma_{11} \\
\kappa_1 \\
\kappa_2 \\
\kappa_3 \\
\kappa'_1
\end{bmatrix}
\]

- VABS can also
  - Deal with trapeze effect, oblique sections
  - Locate neutral axis, principal bending/inertia axes, shear center
  - Recover 3D displacement/strain/stress
  - Model beams made of smart materials (coupled thermo-elasto-electro-magnetic behavior)
What Can VABS Do for You?

- **VABS** takes a finite element discretization of sectional geometry and material as input to calculate sectional properties, which are needed for any beam analysis code to predict global behavior. VABS also recovers 3D displacements/strains/stresses over the section.

- **VABS** can be used independently for structural design of beam sections (topology and material): e.g., maximize torsional stiffness while maintain desired center of gravity.

- **VABS** powers conventional beam elements with the fidelity of 3D detailed FEA for geometry representation and prediction with negligible additional computing time.
VABS: Efficient High-Fidelity Modeling of Composite Slender Structures

- A unique technology continuously funded by US Army since 1988 (28 yrs, 3 more to come).
- Tool of choice for helicopter industry and wind turbine industry.
- Efficient high-fidelity solutions for slender parts: one dimension >> two other dimensions
Related Problems
Related Problems
## Properties of a Wind Turbine Blade

<table>
<thead>
<tr>
<th></th>
<th>PreComp</th>
<th>CROSTAB</th>
<th>VABS</th>
<th>% Diff. (PreComp)</th>
<th>% Diff. (CROSTAB)</th>
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</thead>
<tbody>
<tr>
<td>$E_{I_{22}}$</td>
<td>2.103E+07</td>
<td>1.459E+08</td>
<td>1.916E+07</td>
<td>9.778</td>
<td>661.734</td>
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<td>$E_{I_{33}}$</td>
<td>6.309E+08</td>
<td>4.878E+08</td>
<td>4.398E+08</td>
<td>43.448</td>
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<td>$G_{J}$</td>
<td>1.008E+07</td>
<td>2.469E+07</td>
<td>2.167E+07</td>
<td>53.479</td>
<td>13.950</td>
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<td>$E_{A}$</td>
<td>3.000E+09</td>
<td>2.789E+09</td>
<td>2.387E+09</td>
<td>25.664</td>
<td>16.826</td>
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<tr>
<td>$S_{34}$</td>
<td>-8.132E+06</td>
<td>6.010E+07</td>
<td>1.210E+07</td>
<td>167.204</td>
<td>396.632</td>
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<tr>
<td>$S_{13}$</td>
<td>-1.037E+06</td>
<td>5.216E+08</td>
<td>-2.635E+07</td>
<td>96.065</td>
<td>2.079E+03</td>
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<td>$S_{14}$</td>
<td>-1.301E+08</td>
<td>1.685E+08</td>
<td>-4.724E+08</td>
<td>72.459</td>
<td>135.671</td>
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<tr>
<td>$S_{23}$</td>
<td>-3.776E+05</td>
<td>9.002E+09</td>
<td>-5.222E+04</td>
<td>623.105</td>
<td>1.724E+07</td>
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<tr>
<td>$S_{24}$</td>
<td>8.746E+06</td>
<td>-1.208E+09</td>
<td>1.422E+06</td>
<td>514.904</td>
<td>8.504E+04</td>
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<tr>
<td>$S_{12}$</td>
<td>7.522E+05</td>
<td>-1.723E+09</td>
<td>-3.381E+07</td>
<td>102.225</td>
<td>4.996E+03</td>
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</tbody>
</table>

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
<th>Value</th>
<th>Value</th>
<th>% Diff. (PreComp)</th>
<th>% Diff. (CROSTAB)</th>
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</thead>
<tbody>
<tr>
<td>$\mu$</td>
<td>285.9</td>
<td>289.132</td>
<td>258.053</td>
<td>10.791</td>
<td>12.044</td>
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<tr>
<td>$i_{22}$</td>
<td>2.211</td>
<td>5.144</td>
<td>2.172</td>
<td>1.797</td>
<td>136.837</td>
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<td>$i_{33}$</td>
<td>62.72</td>
<td>61.340</td>
<td>46.418</td>
<td>35.121</td>
<td>32.148</td>
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<tr>
<td>$x_{m2}$</td>
<td>0.332</td>
<td>0.284</td>
<td>0.27780</td>
<td>19.444</td>
<td>2.064</td>
</tr>
<tr>
<td>$x_{m3}$</td>
<td>0.027</td>
<td>-0.028</td>
<td>0.02743</td>
<td>1.572</td>
<td>201.272</td>
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<td>$x_{t2}$</td>
<td>0.331</td>
<td>-0.0290</td>
<td>0.233</td>
<td>42.173</td>
<td>112.466</td>
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<td>$x_{t3}$</td>
<td>0.028</td>
<td>0.2273</td>
<td>0.029</td>
<td>3.287</td>
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<td>$x_{s2}$</td>
<td>0.287</td>
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<td>813.479</td>
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<tr>
<td>$x_{s3}$</td>
<td>0.028</td>
<td>/</td>
<td>0.040</td>
<td>30.478</td>
<td>/</td>
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<tr>
<td>$\theta$</td>
<td>-0.990</td>
<td>3.7919</td>
<td>-1.244</td>
<td>20.419</td>
<td>404.813</td>
</tr>
</tbody>
</table>
Realistic Rotor Blade

- Realistic rotor blade
- 100°C temperature increase
- Find thermal stress
- ANSYS model using brick elements (4.8M DOFs)

Realistic Rotor Blade

\[ \sigma_{11} \]

\[ \sigma_{22} \]

\[ \sigma_{33} \]

\[ \sigma_{23} \]
Realistic Rotor Blade

![Graphs showing normal stresses](image)
4-Layer Laminate Under Torque

\[ T_1 = 1 N \cdot m \]


<table>
<thead>
<tr>
<th>Layup Sequence</th>
<th>( E_{11} ) (GPa)</th>
<th>( E_{22} ) (GPa)</th>
<th>( E_{33} ) (GPa)</th>
<th>( G_{12} ) (GPa)</th>
<th>( G_{13} ) (GPa)</th>
<th>( G_{23} ) (GPa)</th>
<th>( \nu_{12} )</th>
<th>( \nu_{13} )</th>
<th>( \nu_{23} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>[0/90/ ± 45]</td>
<td>132</td>
<td>10.8</td>
<td>10.8</td>
<td>5.65</td>
<td>5.65</td>
<td>3.38</td>
<td>0.24</td>
<td>0.24</td>
<td>0.59</td>
</tr>
</tbody>
</table>
Loss of Accuracy Using Smeared Properties

Graphs showing the comparison of stress values under different conditions.
Loss of Accuracy Using Smeared Properties

Graphs showing the comparison of different methods for calculating stresses ($\sigma_{11}$ and $\sigma_{22}$) as a function of $x_3$ (m). The methods compared are Detailed FEA, VABS, SMP$^1$, and SMP$^2$.
Loss of Accuracy Using Smeared Properties
Elements/Layer Needed for Direct Numerical Simulation

Converged: six 20-noded brick elements/layer thickness.
Conclusion

- **VABS:** an efficient high-fidelity alternative of DNS for slender composite structures.
  - **Best complete set of beam properties:** needed for static/dynamic analysis using beam elements.
  - **Complete set of accurate 3D fields.**
  - **Highly optimized for efficiency:** ply-level details of real blades can be modeled in seconds
  - **Extensively validated** in helicopter and wind industry
  - **Directly integrated** into other design environments
  - **An essential piece for multiscale simulation** of slender composite structures to link properties predicted by micromechanics to structural analysis.