

# State of the Art of Rheology of Concentrated Suspensions

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# Current Projects and Titles-2017

- Numerical simulation of injection molding of long fiber thermoplastic composites (American Chemical Council)
- Benign processing of polymers using water or super- critical carbon dioxide-PAN (ORNL/DOE)
- Generation of sustainable composites based on thermoplastics reinforced with TLCP's, rod-like molecules; automotive applications, and H<sub>2</sub> storage (SRNL/DOE)
- Role of processing on the burst behavior of polyethylene pipes and tubing(Lyondell-Basell).
- High performance materials for use in additive manufacturing/3-D printing(NAI/NASA, 1 position)
- Polymer composites from plants (hemp) (2 positions)
- Novel polymer blends for removal of cancer cells in blood (BioTherapeutics/NIH)

# Outline

- Motivation
  - Long fiber-reinforced plastic composites
  - Mechanical properties and manufacturing
- Background
  - Orientation models
  - Stress tensor
  - Fiber flexibility
  - Rheological testing: Shear and Extension
- Non-lubricated Squeeze Flow
  - Stress Growth
  - Orientation Evolution
- Conclusions & Future Plans

# Objectives

- Develop a rheological test that will induce fiber flexing
  - Allows for testing of semi-flexible models
- Generate experimental stress growth data
  - Ultimate goal is to obtain orientation model parameters through stress-fitting
  - Currently obtain parameters by fitting to orientation data
    - Tedious and labor-intensive

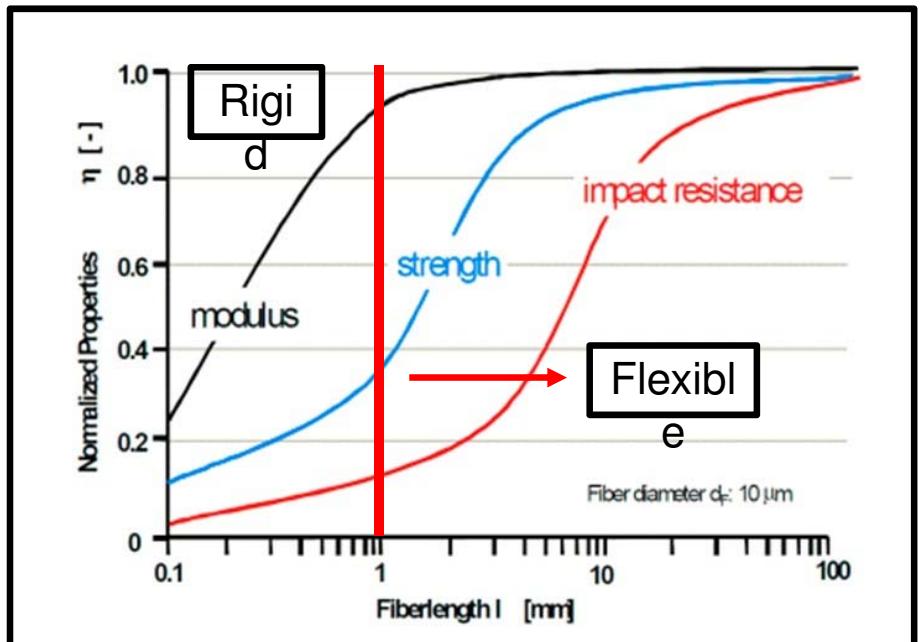
# Motivation: Mechanical Properties

## Orientation Effects

System (glass/epoxy)	Strength ( $10^3$ psi)	Stiffness ( $10^6$ psi)	Volume fraction
Unfilled resin	10-12	0.3-0.4	0
Spherical particles	9-10.5	1.5-1.7	0.50
Short fiber (transverse)	5.5	1.4	0.50
Short fiber (longitudinal)	40	4.5	0.50

Carlson, L.A., "Thermoplastic Composite Materials: Composite Materials Series, Vol 7." Elsevier, NY, 1991.

## Length Effects

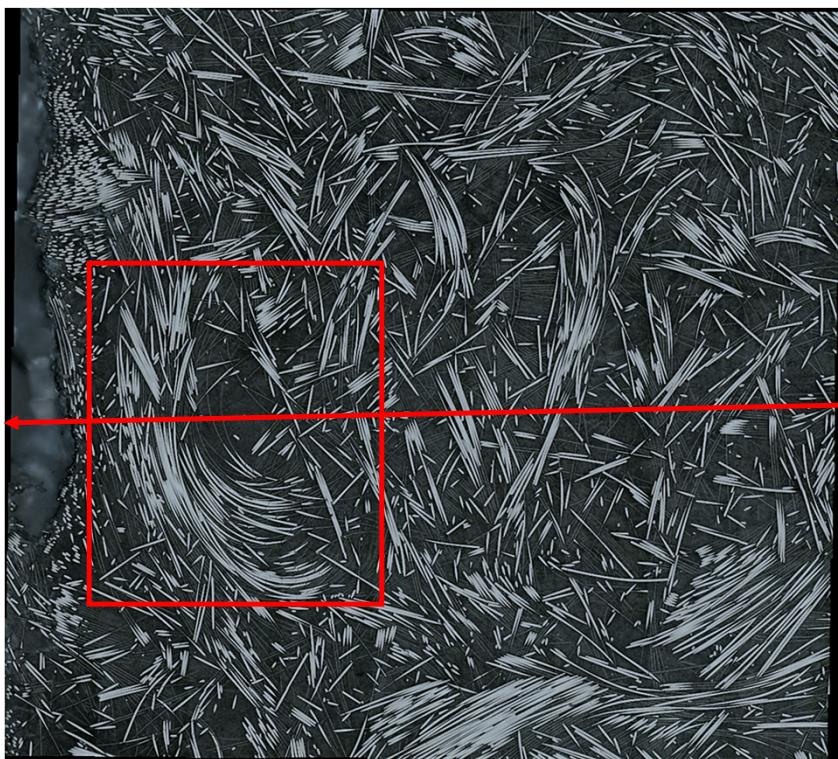


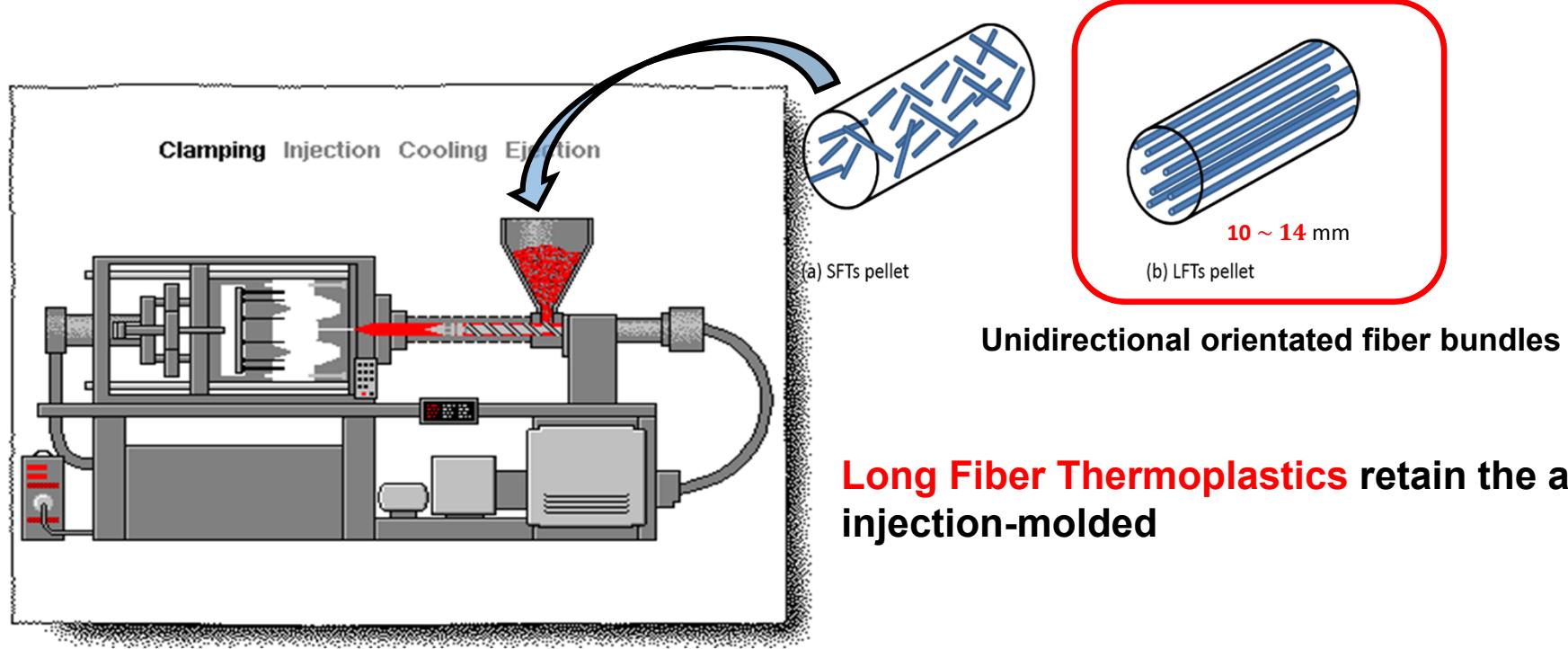
Cieslinski, M., Baird, D. *Progress in Assessing Fiber Orientation and Flexibility with Increased Fiber Lengths*. ANTEC 2015. 23-25 March 2015.

# Top Down

50 wt% Nylon 6,6

0% W and 10% L





Unidirectional orientated fiber bundles

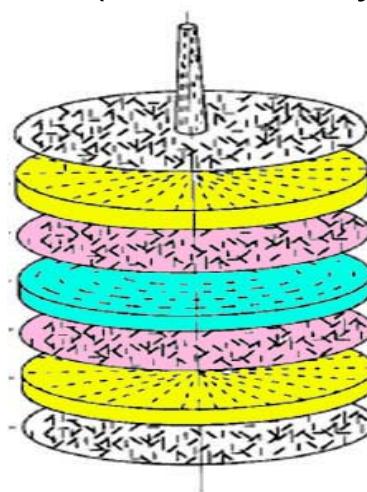
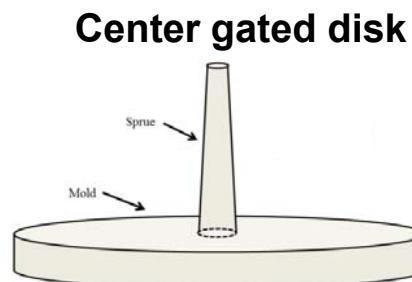
**Long Fiber Thermoplastics** retain the ability to be injection-molded

## Widely Used in Industry & Suitable for Fiber Thermoplastic Composites

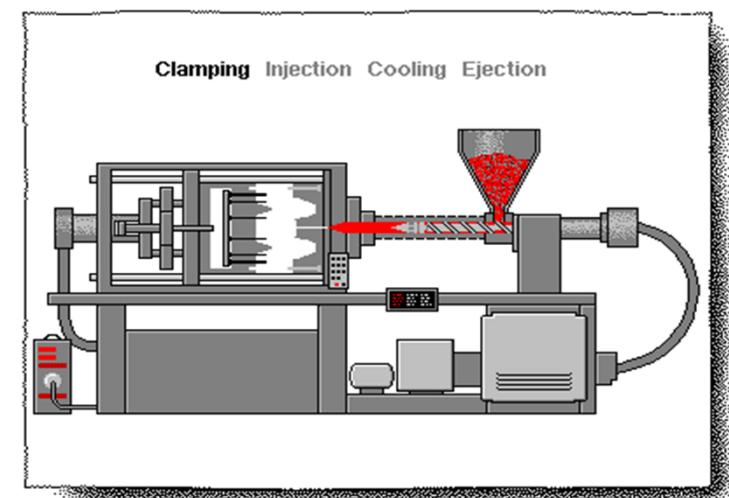
- Rapid & Automatic
- Repeatability & Geometrical Complexity

## Two issues/facts of IM Long Fiber Thermoplastic Composites

- Flow induced variable orientation (Mold Cavity)



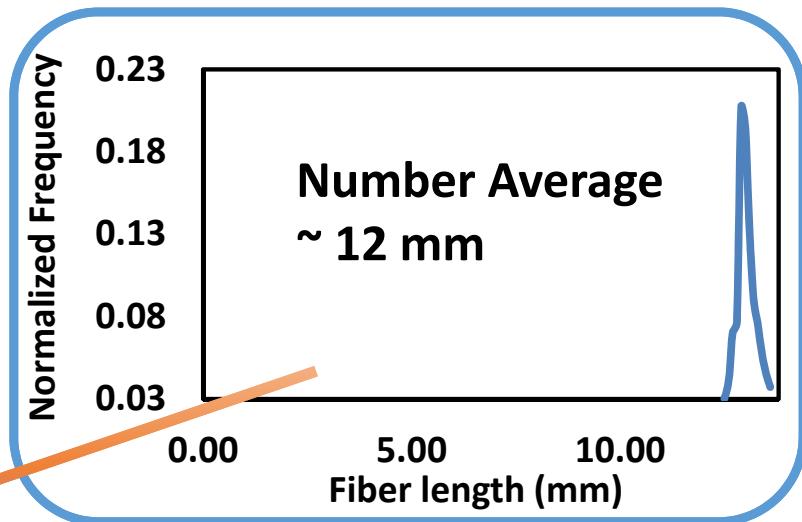
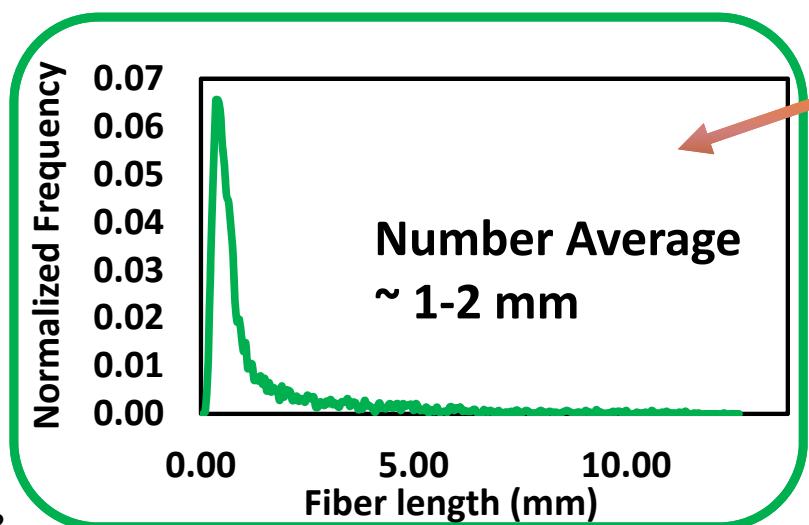
3~4 distinguishable layers:



Region	Orientation	Source
Skin	Random in $r\theta$ plane	Thermal + Fountain
Shell	Flow Aligned	Shear flow
Transition	No preferential	Shear & extension
Core	Transverse to flow	Extensional flow
Transition	No preferential	Shear & extension
Shell	Flow Aligned	Shear
Skin	Random in $r\theta$ plane	Thermal + Fountain

# Factors Affecting Properties in Injection Molding

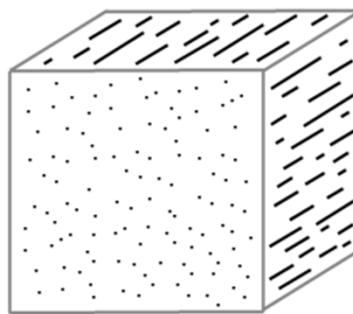
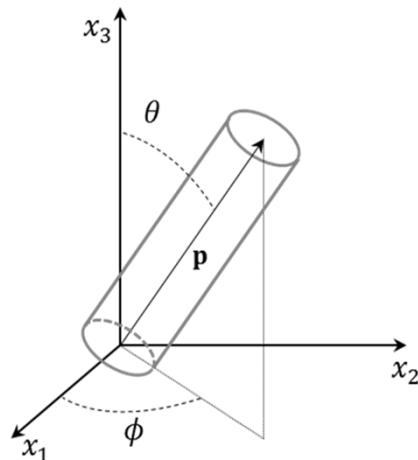
## □ Fiber Breakage (Broad Distribution)



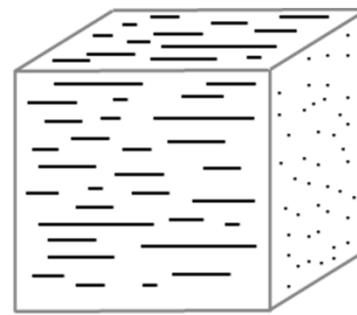
Nguyen, 2008

Huq and Azaiez, 2005

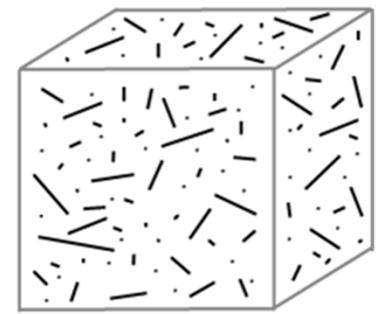
# Background: Orientation



$$\mathbf{A} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$$



$$\mathbf{A} = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{pmatrix}$$



$$\mathbf{A} = \begin{pmatrix} 1/3 & 0 & 0 \\ 0 & 1/3 & 0 \\ 0 & 0 & 1/3 \end{pmatrix}$$

$$\mathbf{A} = \int \mathbf{p} \mathbf{p} \psi(\mathbf{p}, t) d\mathbf{p}$$

$$\mathbf{A}_4 = \int \mathbf{p} \mathbf{p} \mathbf{p} \mathbf{p} \psi(\mathbf{p}, t) d\mathbf{p}$$

# Background: Orientation Dynamics

$$\frac{DA}{Dt} = \alpha((W \cdot A - A \cdot W) + \xi(D \cdot A + A \cdot D - 2D:A_4) + 2C_I \dot{\gamma}(I - 3A))$$

Matrix  
Contribution

Isotropic  
Rotary  
Diffusion

## Empirical Parameters

$\alpha$	Fiber slip relative to matrix
$C_I$	Fiber interactions

Folgar, F. and C.L. Tucker III, *Orientation behavior of fibers in concentrated suspensions*. Journal of Reinforced Plastics and Composites, 1984. 3(2): p. 98-119.

Huynh, H.M., *Improved Fiber Orientation Predictions for Injection-Molded Composites*. 2001, University of Illinois at Urbana-Champaign.

# Semi-Flexible Fibers

$$\frac{D\mathbf{A}}{Dt} = \alpha \left[ \mathbf{W} \cdot \mathbf{A} - \mathbf{A} \cdot \mathbf{W} + \xi (\mathbf{D} \cdot \mathbf{A} + \mathbf{A} \cdot \mathbf{D} - 2\mathbf{D} : \mathbf{A}_4) + 2C_I \dot{\gamma} (\mathbf{I} - 3\mathbf{A}) + \frac{l_B}{2} [\mathbf{Cm} + \mathbf{mC} - 2(\mathbf{m} \cdot \mathbf{C})\mathbf{A}] + 2k(\mathbf{B} - \mathbf{A} tr(\mathbf{B})) \right]$$

$$\frac{D\mathbf{B}}{Dt} = \alpha \left[ \mathbf{W} \cdot \mathbf{B} - \mathbf{B} \cdot \mathbf{W} + \xi (\mathbf{D} \cdot \mathbf{B} + \mathbf{B} \cdot \mathbf{D} - 2\mathbf{D} : \mathbf{B}) - 4C_I \dot{\gamma} \mathbf{B} + \frac{l_B}{2} [\mathbf{Cm} + \mathbf{mC} - 2(\mathbf{m} \cdot \mathbf{C})\mathbf{B}] + 2k(\mathbf{A} - \mathbf{B} tr(\mathbf{B})) \right]$$

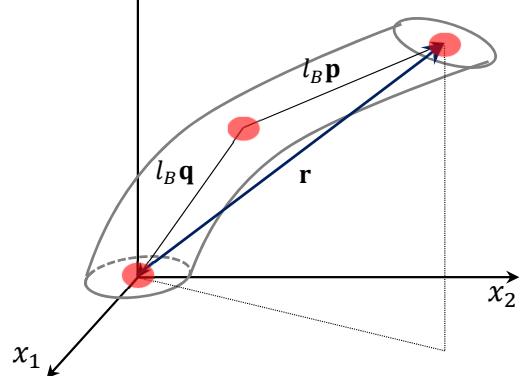
$$\frac{D\mathbf{C}}{Dt} = \alpha \left[ \nabla \mathbf{v}^t \cdot \mathbf{C} - (\mathbf{A} : \nabla \mathbf{v}^t) \mathbf{C} - 2C_I \dot{\gamma} \mathbf{C} + \frac{l_B}{2} [\mathbf{m} - \mathbf{C}(\mathbf{m} \cdot \mathbf{C})] - k\mathbf{C}(1 - tr(\mathbf{B})) \right]$$

Hydrodynamic

IRD

Bending  
From Flow

Bending  
Potential



$$\mathbf{A} = \iint \mathbf{p} \mathbf{p} \psi(\mathbf{p}, \mathbf{q}, t) d\mathbf{p} d\mathbf{q}$$

$$\mathbf{B} = \iint \mathbf{p} \mathbf{q} \psi(\mathbf{p}, \mathbf{q}, t) d\mathbf{p} d\mathbf{q}$$

$$\mathbf{C} = \iint \mathbf{p} \psi(\mathbf{p}, \mathbf{q}, t) d\mathbf{p} d\mathbf{q}$$

$$\mathbf{A}_4 = \iint \mathbf{p} \mathbf{p} \mathbf{p} \mathbf{p} \psi(\mathbf{p}, \mathbf{q}, t) d\mathbf{p} d\mathbf{q}$$

$$\mathbf{m} = \sum_{i=1}^3 \sum_{j=1}^3 \sum_{k=1}^3 \frac{\partial^2 v_i}{\partial x_j \partial x_k} A_{jk} \mathbf{e}_i$$

$$\mathbf{r} = l_B (\mathbf{p} - \mathbf{q})$$

$$\mathbf{R} = \frac{\langle \mathbf{r} \mathbf{r} \rangle}{tr(\mathbf{r} \mathbf{r})} = \frac{\mathbf{A} - \mathbf{B}}{1 - tr(\mathbf{B})}$$

Strautins and Latz, 2007

Ortman et al., 2012

# Coupling Orientation to Flow

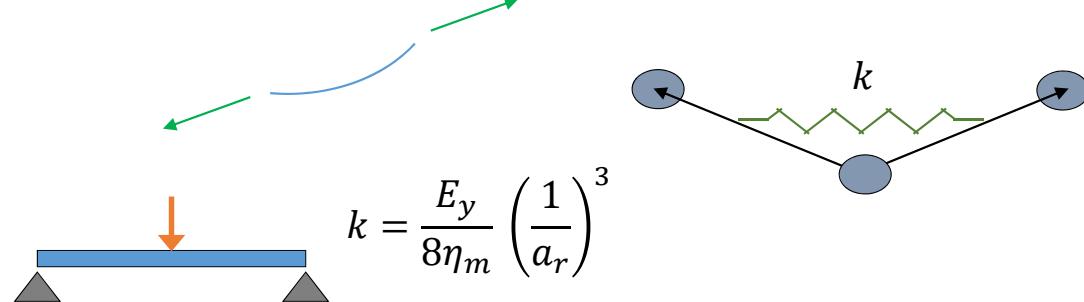
Stress Equation for Rigid Fibers:

$$\boldsymbol{\sigma} = -P\mathbf{I} + 2\eta_m \mathbf{D} + 2\eta_m \phi (\mu_1 \mathbf{D} + \mu_2 \mathbf{D} : \mathbf{A}_4)$$

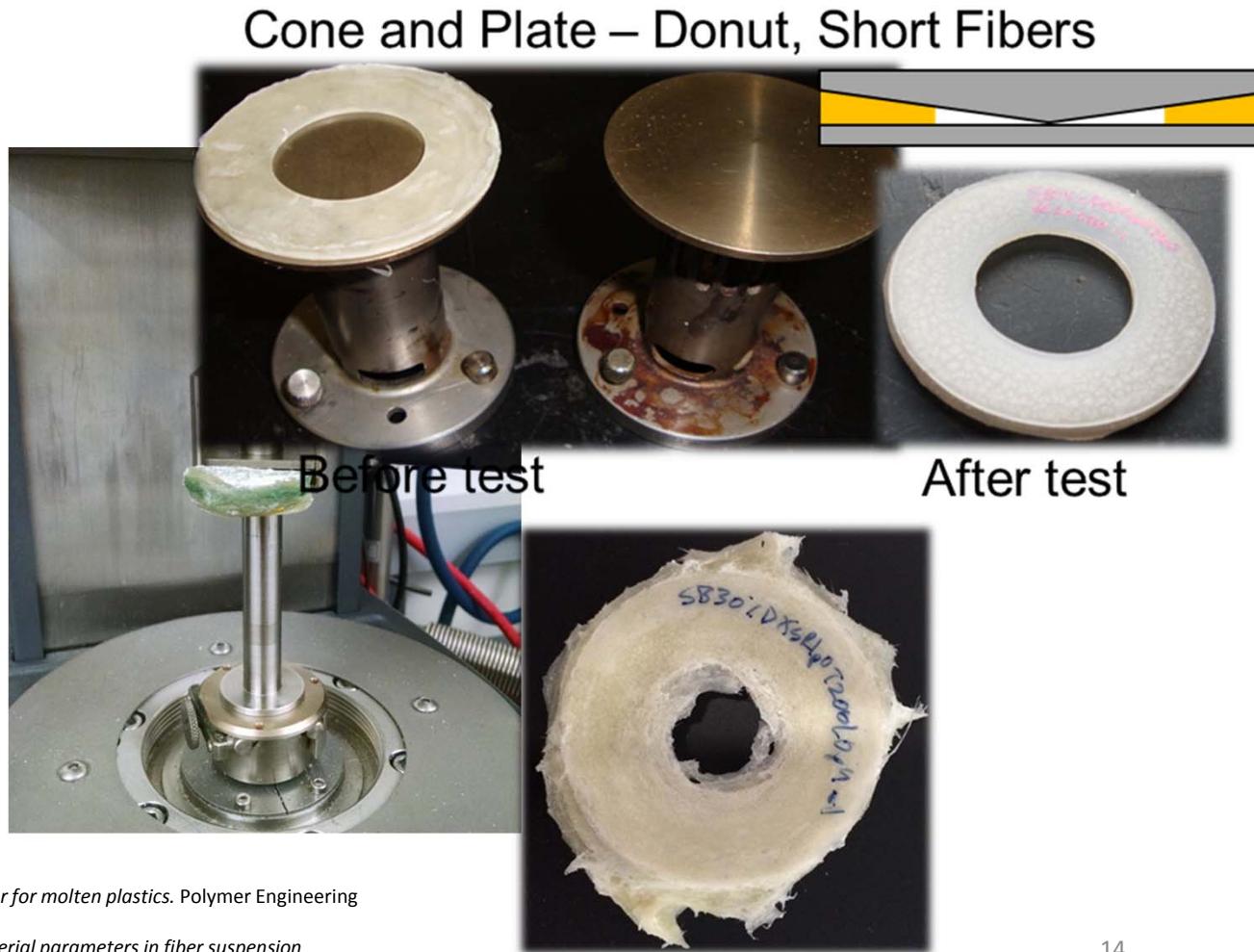
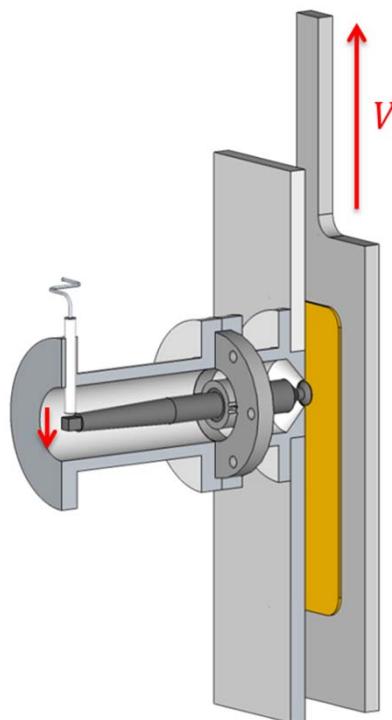


Proposed Stress Equation for Semi-Flexible Fibers:

$$\boldsymbol{\sigma} = -P\mathbf{I} + 2\eta_m \mathbf{D} + 2\eta_m \phi (\mu_1 \mathbf{D} + \mu_2 \mathbf{D} : \mathbf{R}_4) + \eta_m k \frac{3\phi a_r}{2} (\mathbf{B} - \mathbf{A} tr \mathbf{B})$$



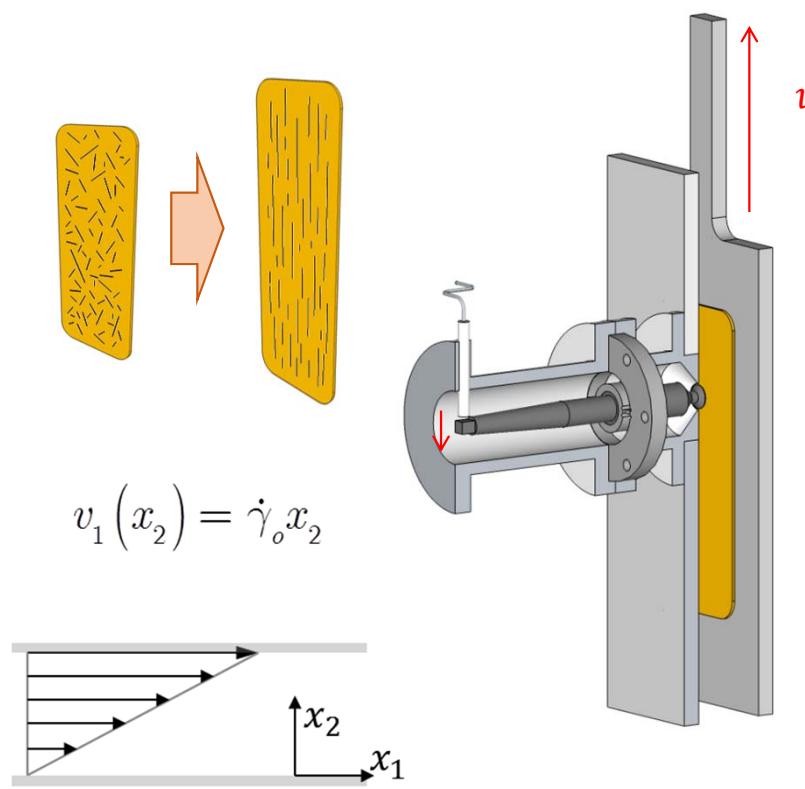
# Experiments



Oakley, J.G. and A.J. Giacomin, *A sliding plate normal thrust rheometer for molten plastics*. Polymer Engineering and Science, 1994. **34**(7): p. 580-4.

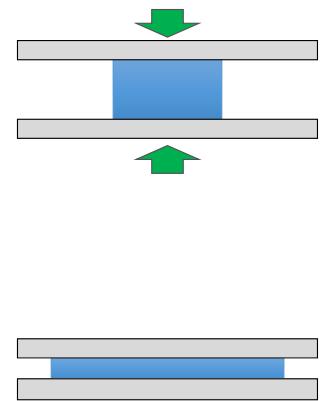
Eberle, A.P.R., et al., *Using transient shear rheology to determine material parameters in fiber suspension theory*. Journal of Rheology, 2009. **53**(3): p. 685-705.

# Model Parameter Obtaining



Startup of Simple Shear

Shear-free Flow  
(Lubricated Squeeze Flow)



Giacomin, 1987  
Dealy and Soong, 1984

$$\nabla \cdot \mathbf{v} = 0 \quad (\text{Continuity})$$

$$\nabla \cdot \boldsymbol{\sigma} = 0 \quad (\text{Momentum})$$

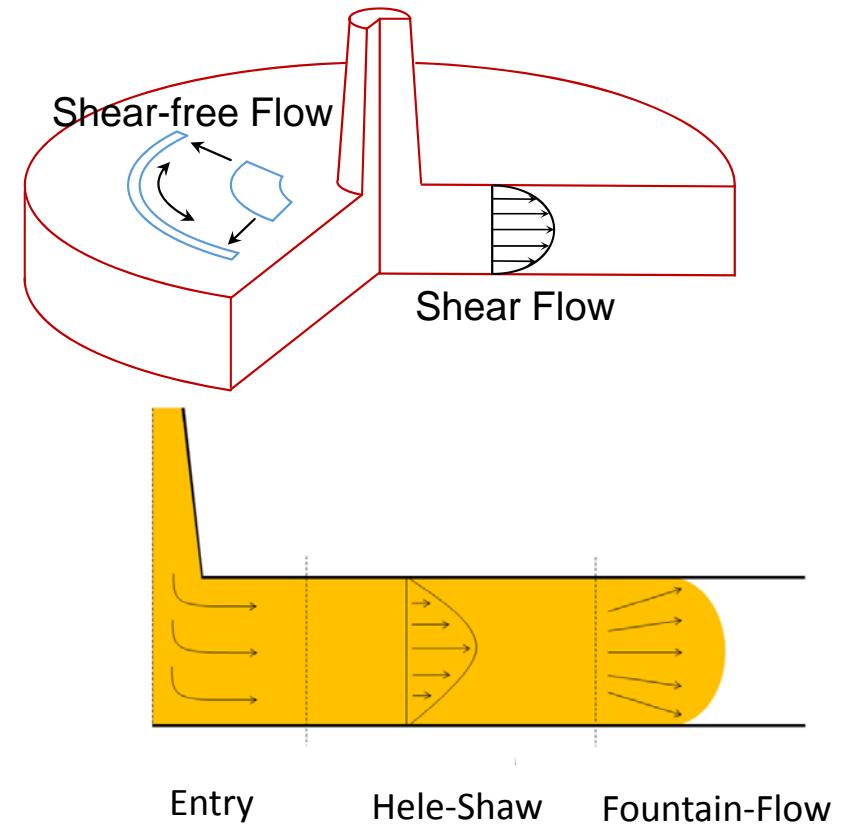
$$\boldsymbol{\sigma} = -P\mathbf{I} + 2\eta_m \mathbf{D} \quad (\text{Stress})$$

$$\frac{\partial c}{\partial t} + \mathbf{v} \cdot \nabla c = 0 \quad (\text{Pseudo-concentration})$$



Decoupled

$$\frac{\partial \mathbf{A}}{\partial t} + \mathbf{v} \cdot \nabla \mathbf{A} - c \mathbf{F}(\nabla \mathbf{v}, \mathbf{A}) = 0$$

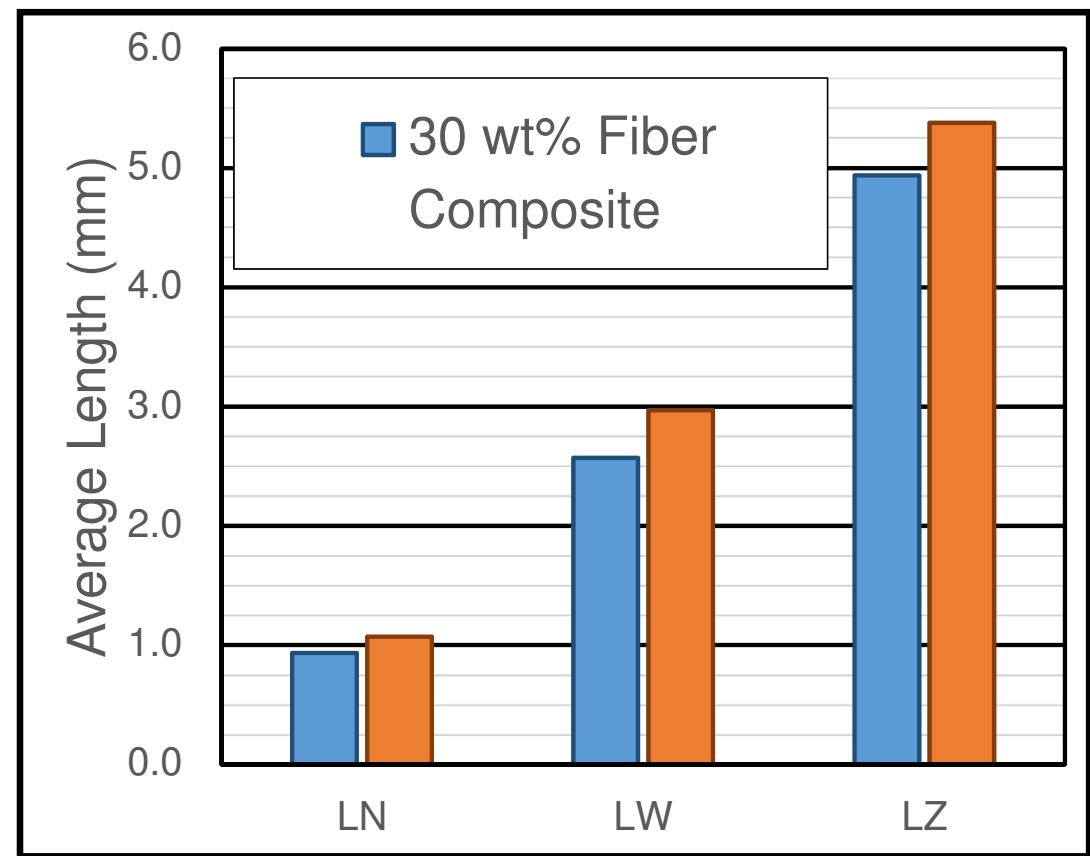


# Nonlubricated Squeeze Flow

$$L_N = \frac{\sum N_i L_i}{\sum N_i}$$

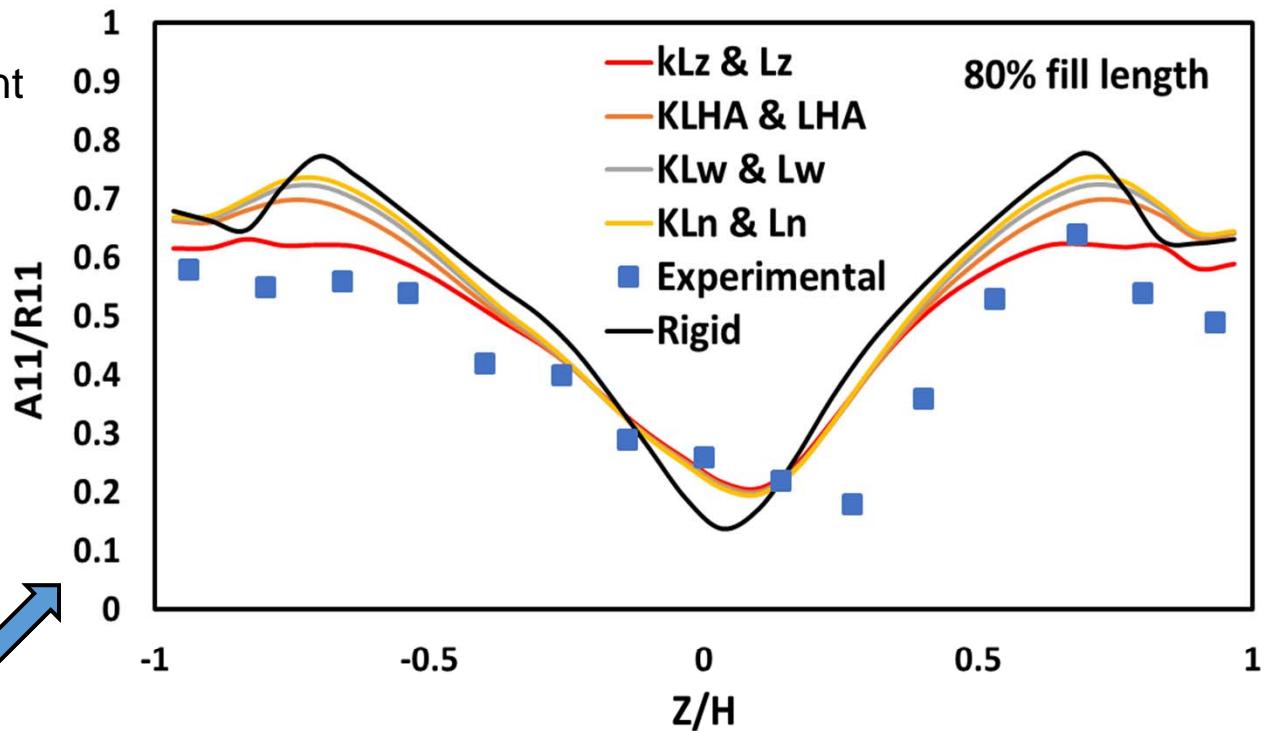
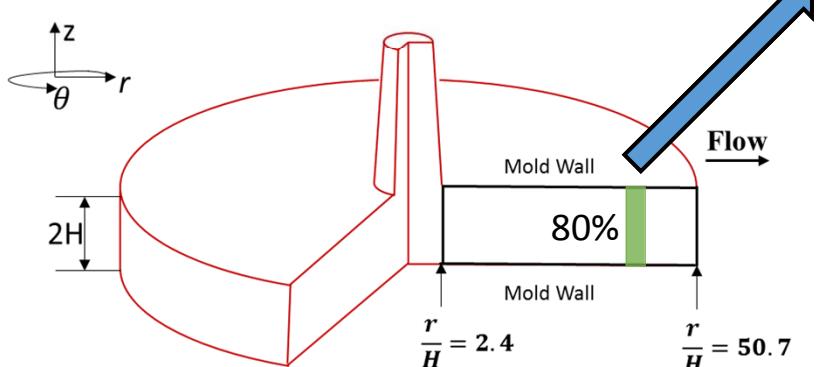
$$L_W = \frac{\sum N_i L_i^2}{\sum N_i L_i}$$

$$L_Z = \frac{\sum N_i L_i^3}{\sum N_i L_i^2}$$



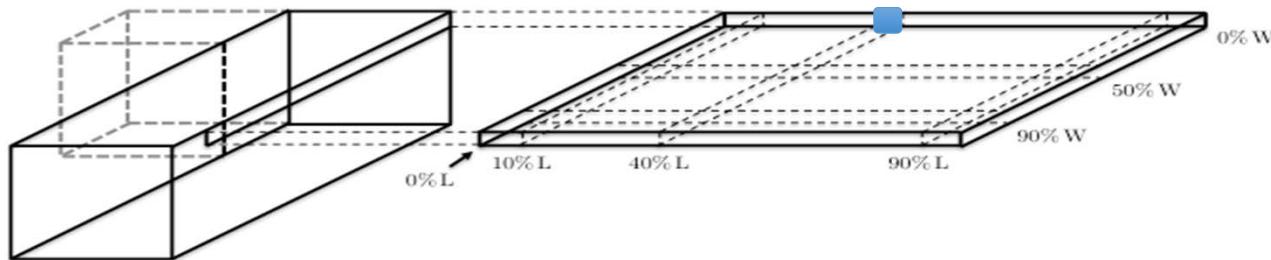
# Flow Direction Fiber Orientation

Location Close to the Advancing Front



- ❑ Both models predict noticeable drops of the values near the wall due to the fountain flow effect.
- ❑ Bead-Rod model shows improvement over the rigid model especially when the longest length parameters are used.

## LGF Orientation Predictions in a EGP Parameters



Fitted to Experimental Data (Solid):

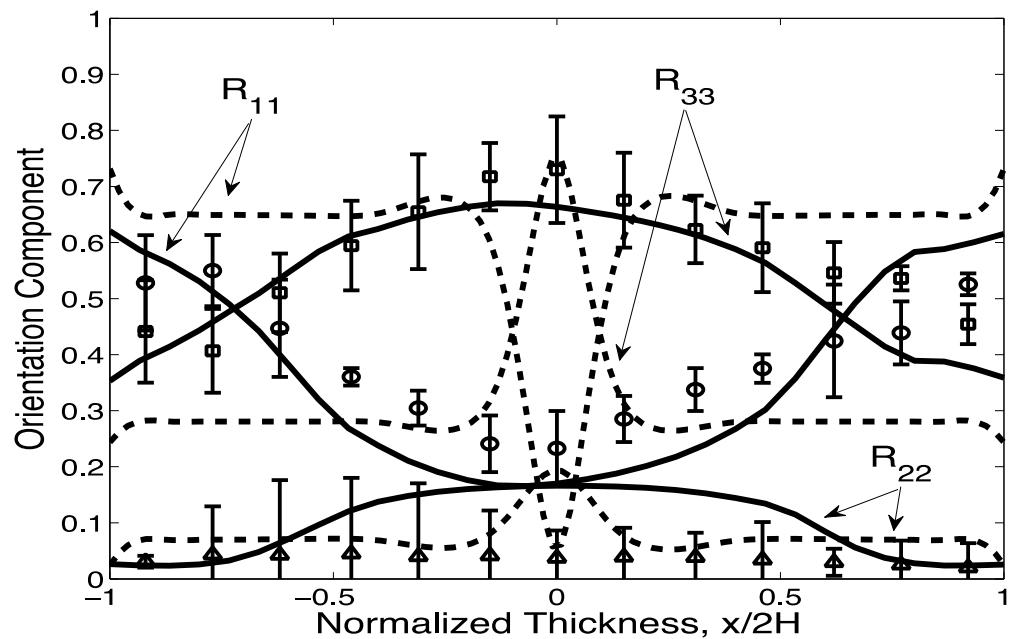
$$\alpha = 0.0039$$

$$C_I = 0.4843$$

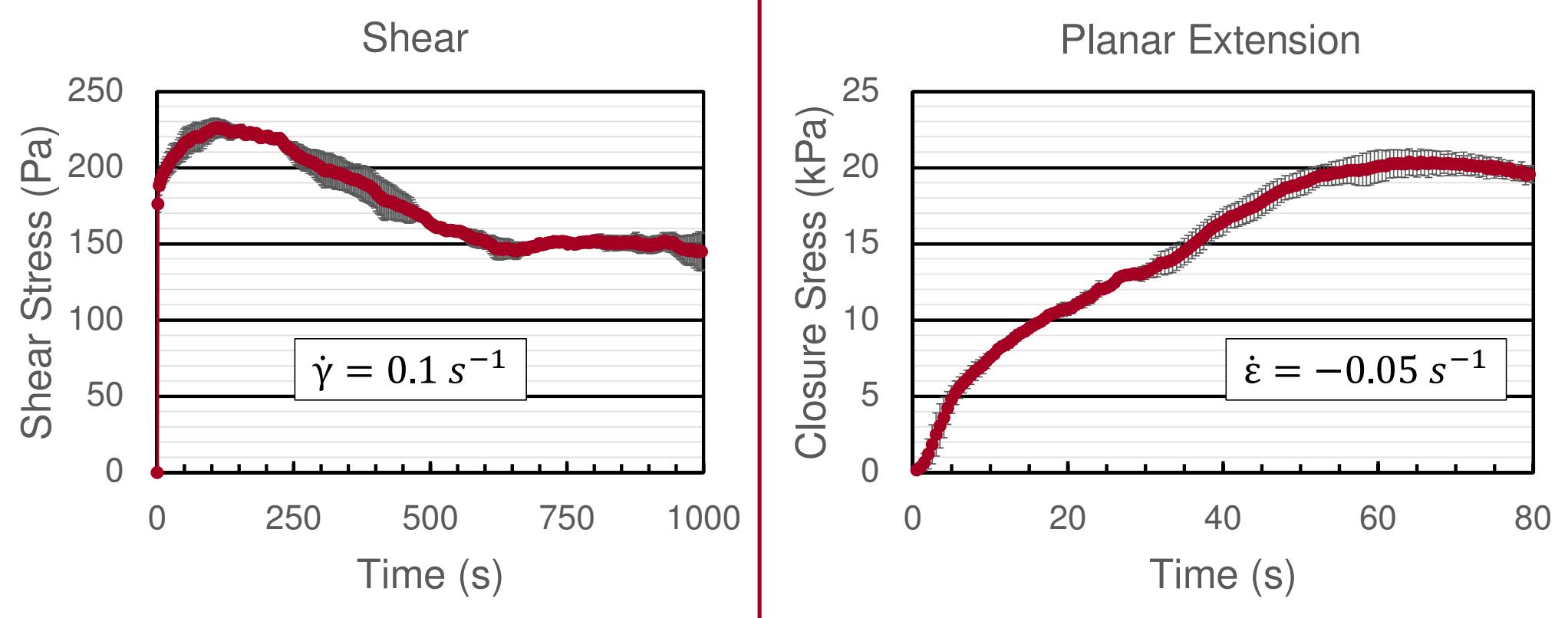
Fitted to Rheology (Dashed):

$$\alpha = 0.13$$

$$C_I = 0.0530$$

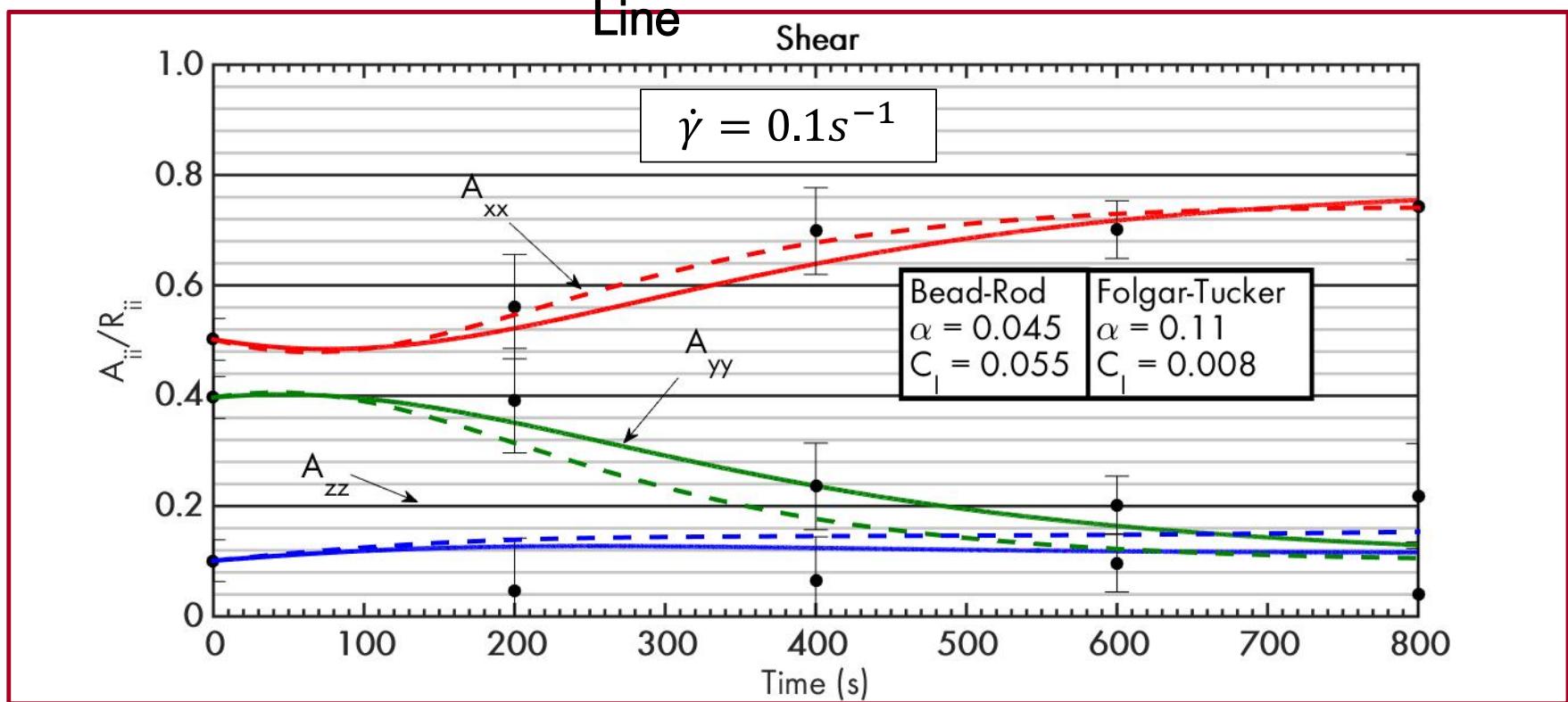


# Experiments



# Experiments

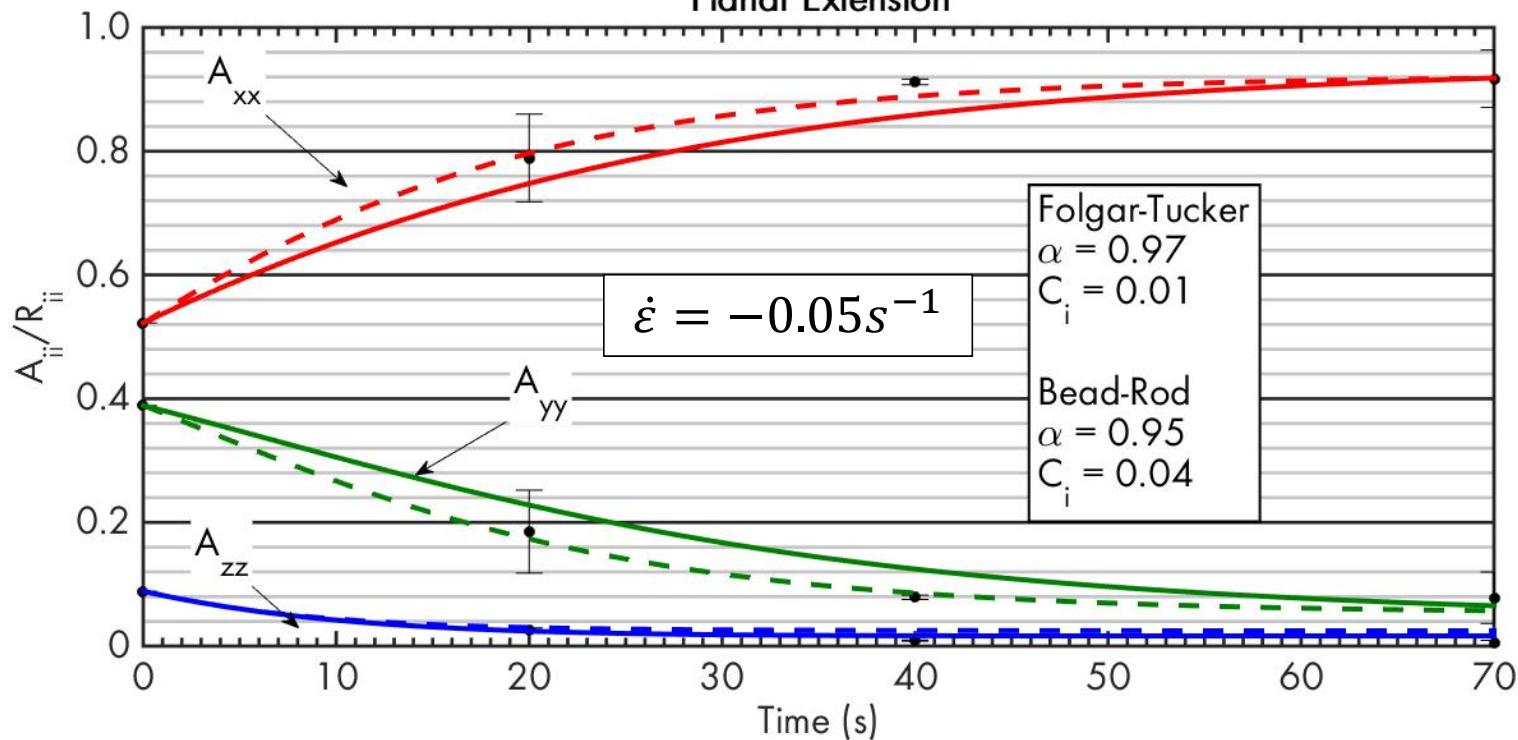
Rigid: Solid Line  
Flexible: Dashed  
Line



# Experiments

Rigid: Solid Line  
Flexible: Dashed  
Line

Planar Extension



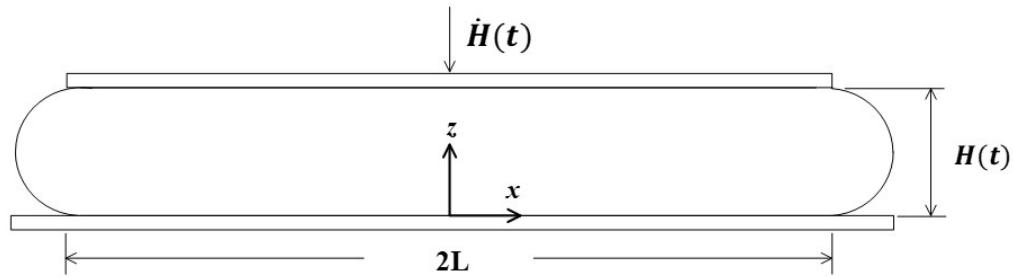
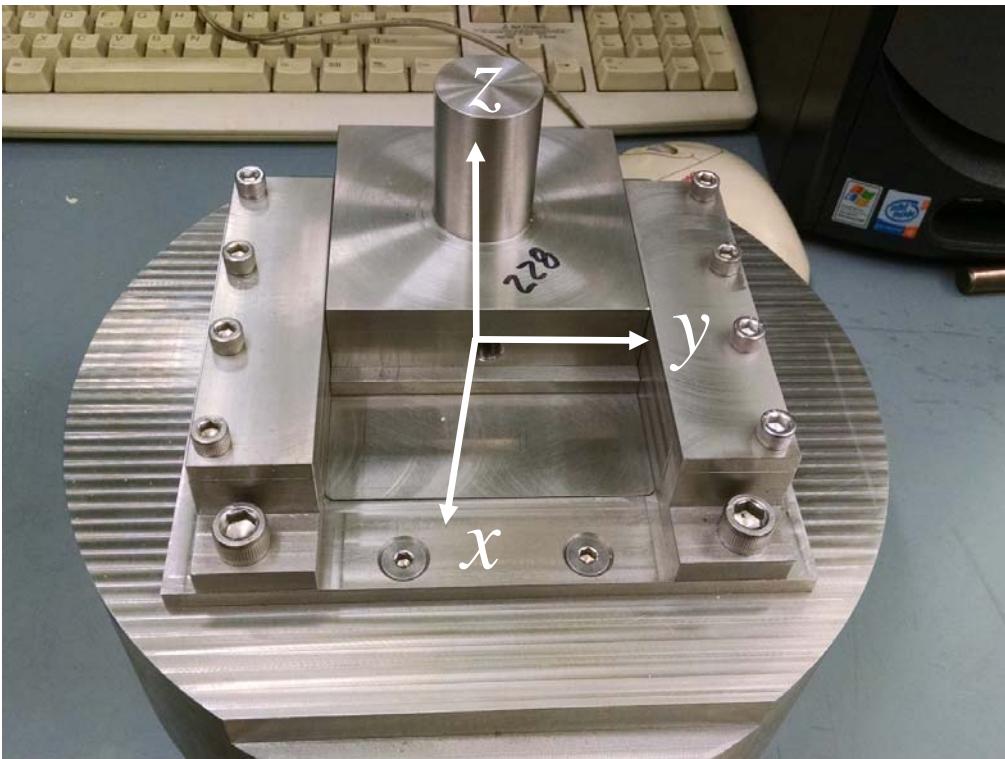
# Background: Empirical Parameters

	<b>Parameter</b>	<b>Shear</b>	<b>Extension</b>
Rigid	$\alpha$	0.11	0.97
	$C_l$	0.008	0.01
Flexible	$\alpha$	0.045	0.95
	$C_l$	0.055	0.04

# Ongoing and Future Efforts

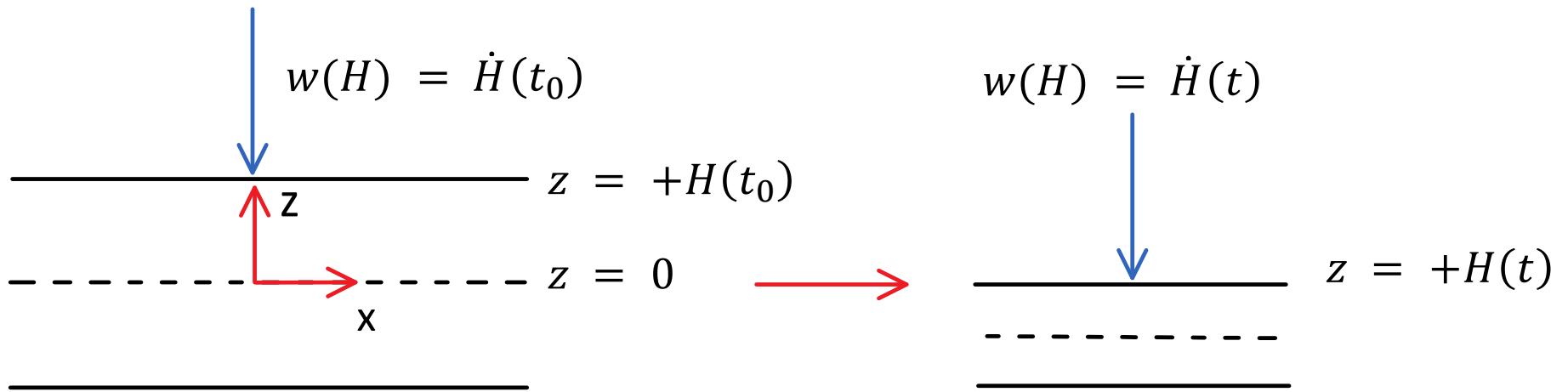
- Develop a test that will induce fiber flexing
  - Allows for testing of semi-flexible models
- Generate experimental stress growth data
  - Ultimate goal is to obtain orientation model parameters through stress-fitting
  - Currently obtain parameters by fitting to orientation data
    - Tedious and labor-intensive
    - Identifying bending parameter through stress relaxation tests

# Experimental: NLSF



- Combination of shear and extension
- Second-order velocity gradients
- Closure stress easily measured

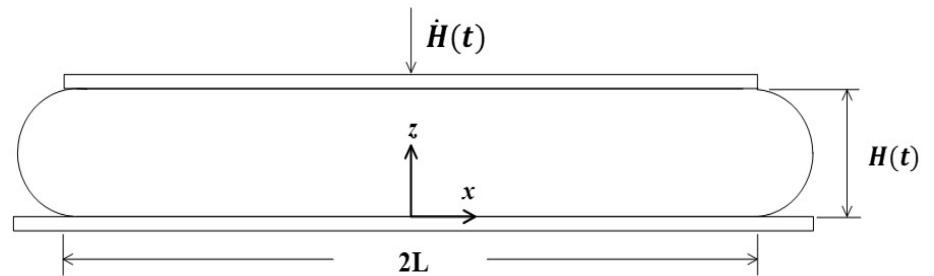
# NLSF Schematic



# Experimental: NLSF

$$u(x, z, t) = -6 \frac{\dot{H}}{H} x \left[ \left( \frac{z}{H} \right) - \left( \frac{z}{H} \right)^2 \right]$$

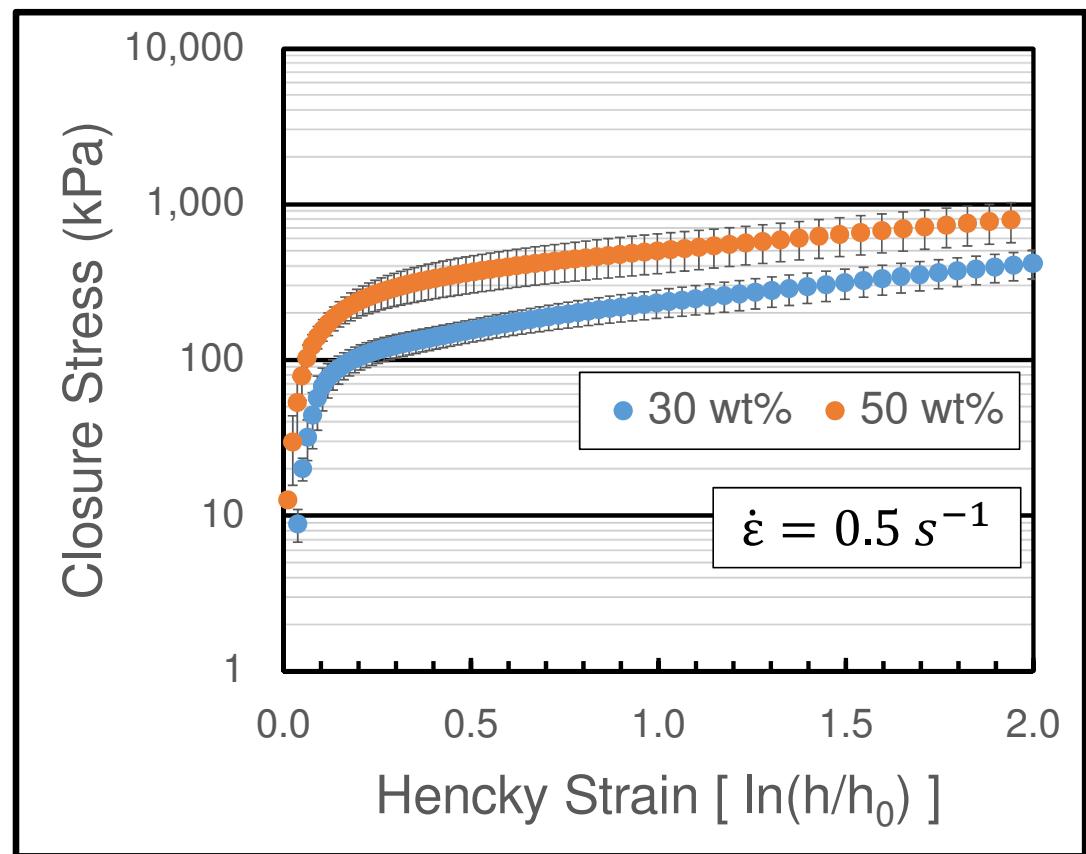
$$w(z, t) = \dot{H} \left[ 3 \left( \frac{z}{H} \right)^2 - 2 \left( \frac{z}{H} \right)^3 \right]$$



$$P(x, z, t) = 6\eta \frac{\dot{H}}{H} \left[ \frac{x^2}{H^2} + \frac{z}{H} - \frac{z^2}{H^2} \right] + P_a$$

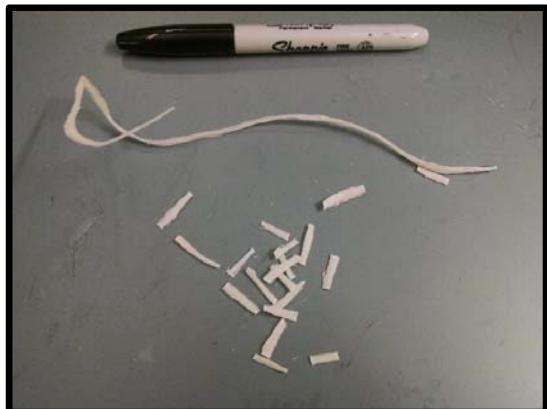
## Nonlubricated Squeeze Flow

- Stress increases with fiber content
- Similar behavior in each case
- Increase from zero
  - GNF-based stress models cannot predict this



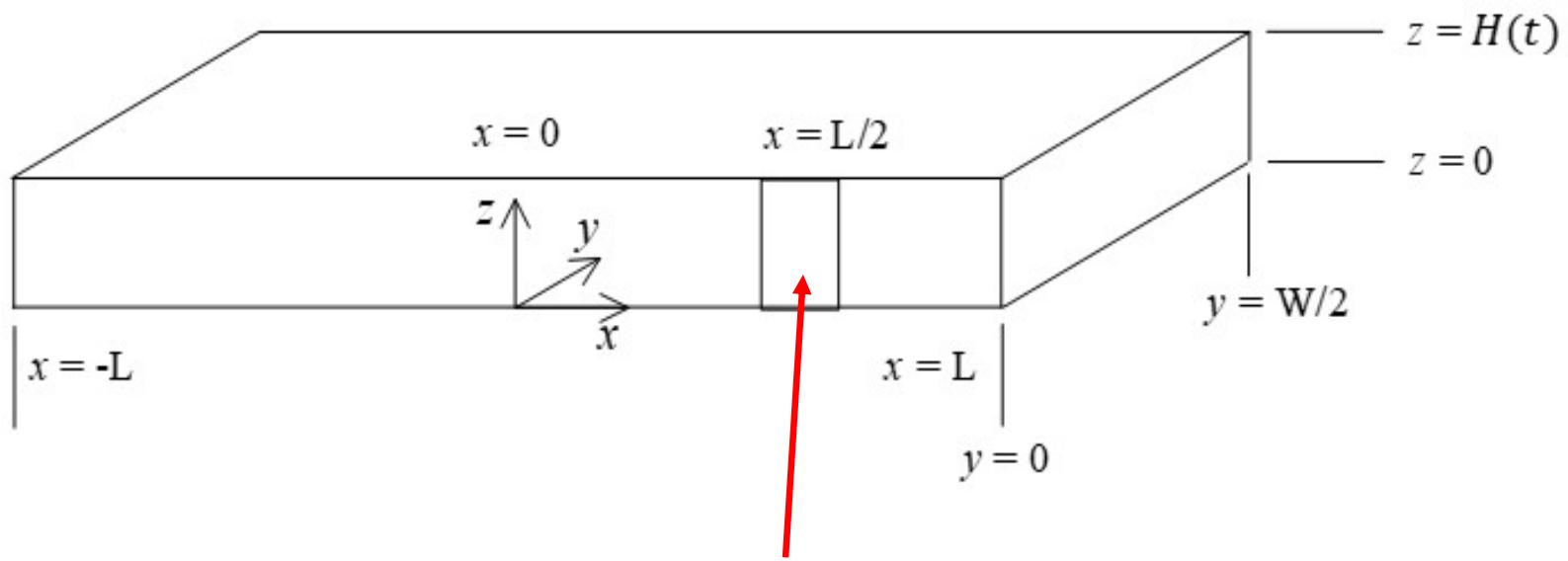
❖ Error bars represent 95%  
CI

# Experimental: Sample Prep



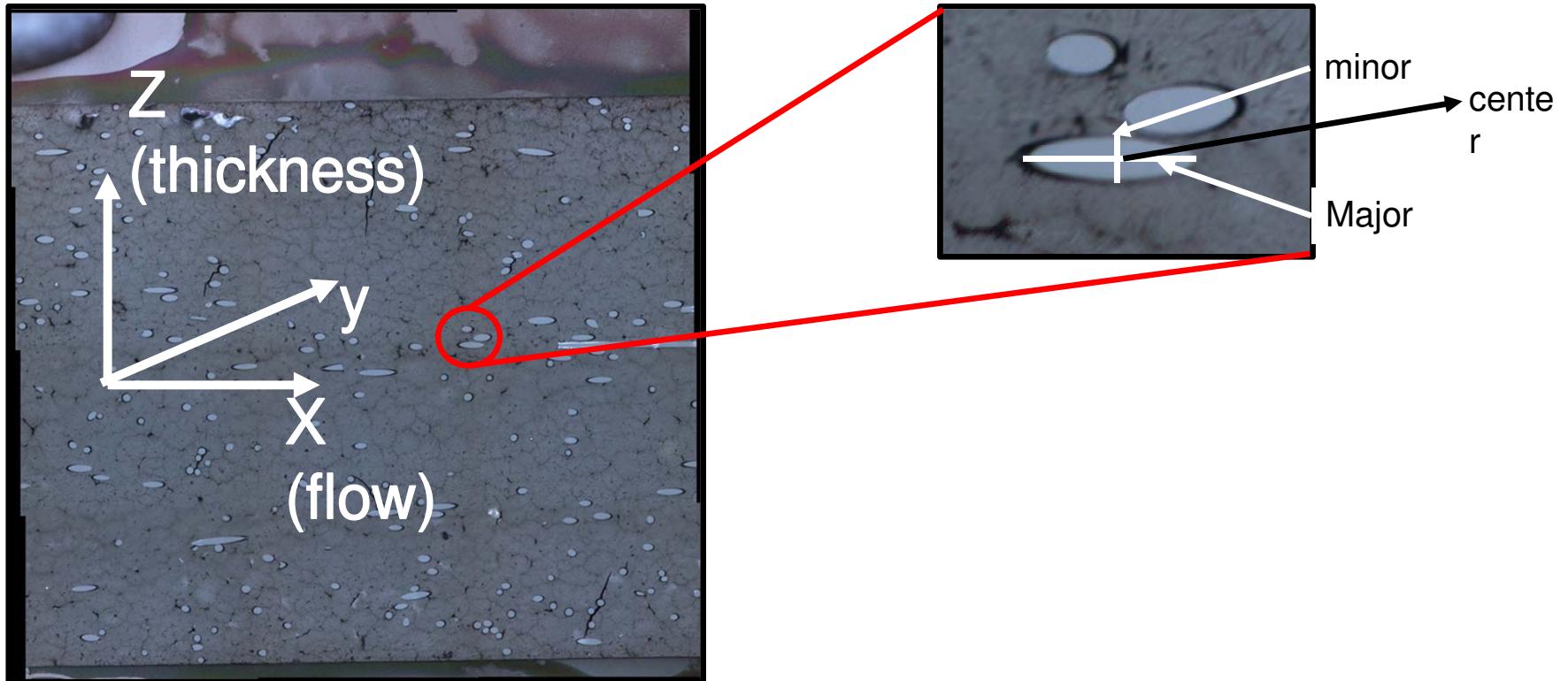
- Samples made of nozzle purge produced using the same conditions as injection-molded CGDs
- Testing Temperature: 200°C
- Constant Hencky Strain Rate: -0.50 s<sup>-1</sup>
- Sample Dimensions
  - 3.75 in (95.25 mm) wide
  - 2 in (50.8 mm) long
  - 7.50 mm thick
- Initial planar random fiber orientation
  - Compression molded “unidirectional” strands
- 30 wt% Short Glass Fiber + Polypropylene (SABIC)

# Experimental: Sample Prep



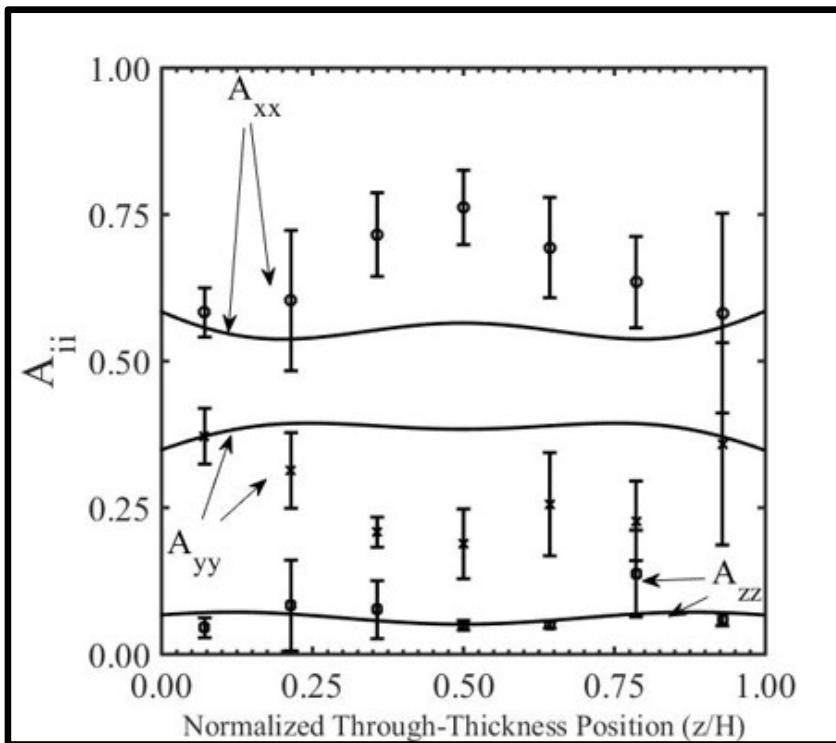
Through-Thickness  
Orientation Observation

# Experimental: Sample Prep

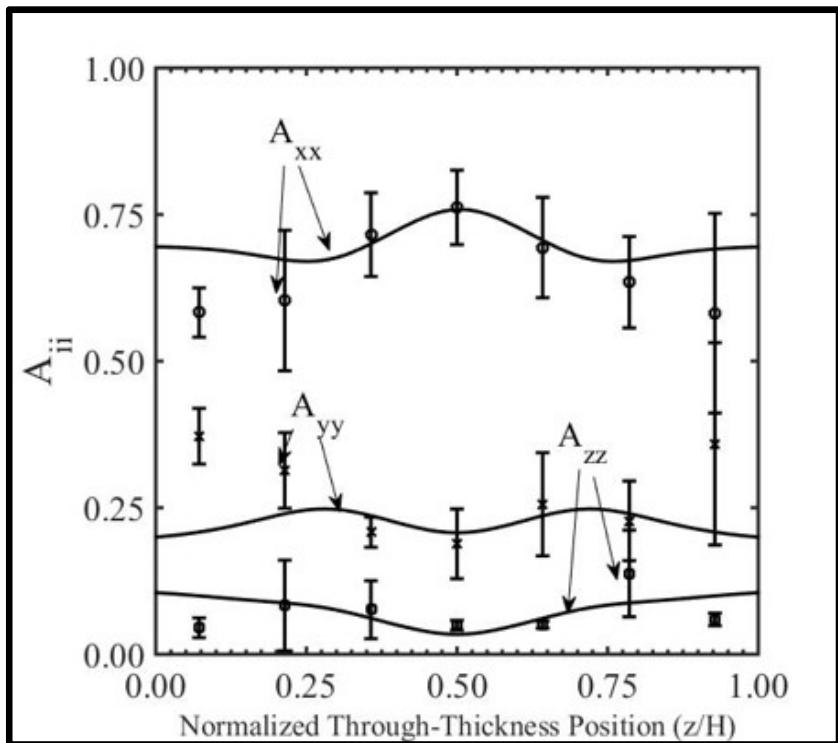


# Results: Orientation

Parameters from Startup of Simple Shear



Parameters from NLSF



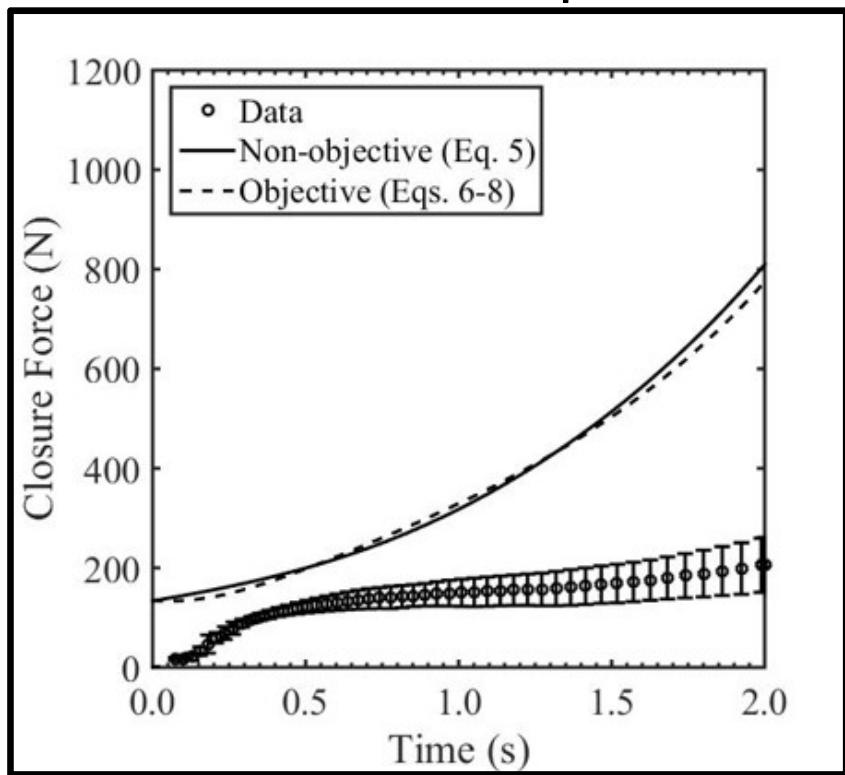
# Results: Orientation

Parameter	Cieslinski et al. <sup>1</sup> (simple shear)	NLSF <sup>2</sup>
a	0.20	1.00
C <sub>I</sub>	0.005	0.020

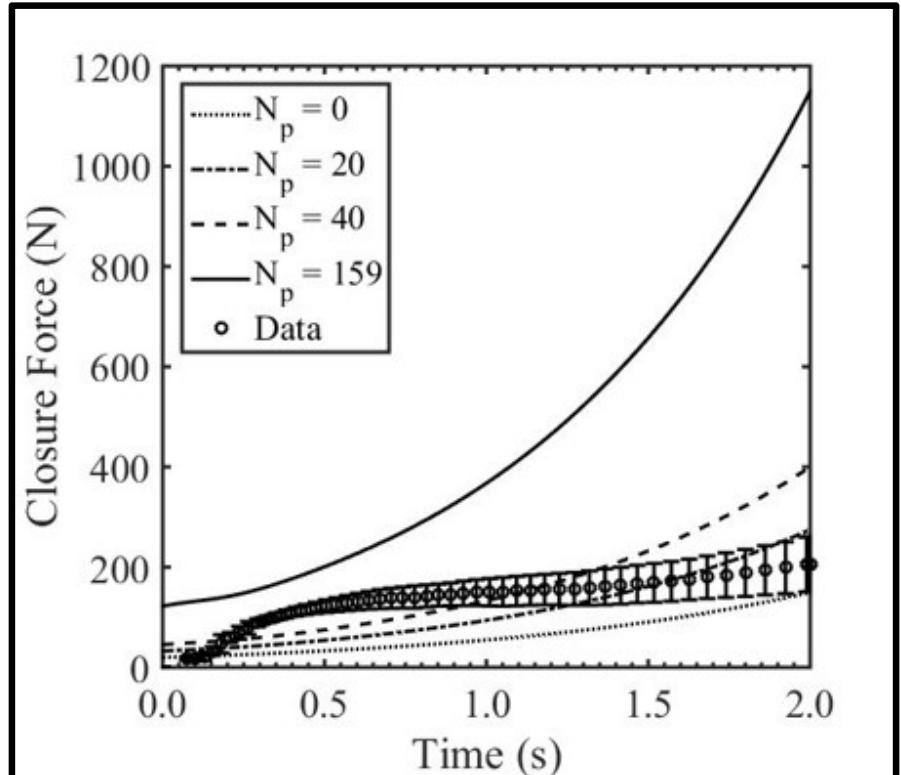
1. Cieslinski, M.J., P. Wapperom, and D.G. Baird, *Fiber orientation evolution in simple shear flow from a repeatable initial fiber orientation*. J. Non-Newton. Fluid Mech., 2016. 237: p. 65-75.
2. Lambert, G.M. et al. *Obtaining short fiber orientation model parameters using non-lubricated squeeze flow*. Phys. Fluids, 2017 (under review)

# Results: Stress Growth

Parameters from Simple Shear



Parameters from NLSF



# Conclusions

- Parameters in orientation models obtained from planar extension are different from those in shear flow
- These parameters lead to a significant difference in the prediction of orientation distribution in an injection molded disk (center-gated) especially in the semi-flexible fiber model
- Homogeneous flows tend to not test the bending contribution to stress and fiber orientation
- Non-lubricated squeeze flow will potentially lead to a method for obtaining the parameters in the orientation models from basic flow properties
- Existing stress tensors have a flaw in the startup of flow which needs to be addressed.

# Future Work

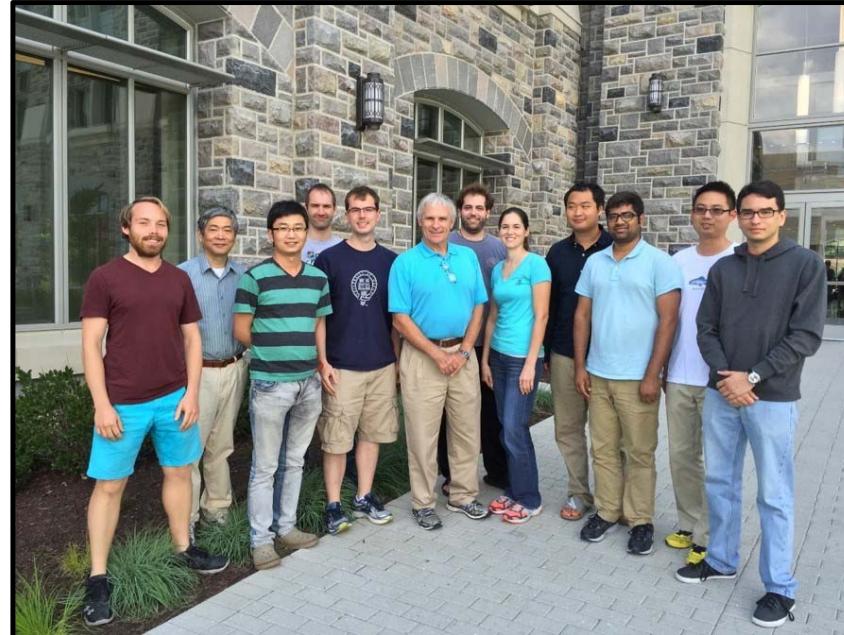
- Obtain orientation data on samples subjected to non-lubricated squeeze flow
- Obtain orientation parameters in the Bead-Rod Model from this orientation data
- Compare the values obtained above with those obtained from fitting stress growth data and possibly stress relaxation data
- Use wet-layed prepared samples to control fiber length and minimize fiber breakage

# Acknowledgements

EASTMAN

سابك  
sabic

American  
Chemistry  
Council



Macromolecules  
Innovation Institute  
*At the intersection of science, engineering, and society*

OAK RIDGE  
National Laboratory



Go Further

Pacific Northwest  
NATIONAL LABORATORY  
Proudly Operated by Battelle Since 1965

U.S. DEPARTMENT OF  
ENERGY