Master stiffness and strength of composite laminates

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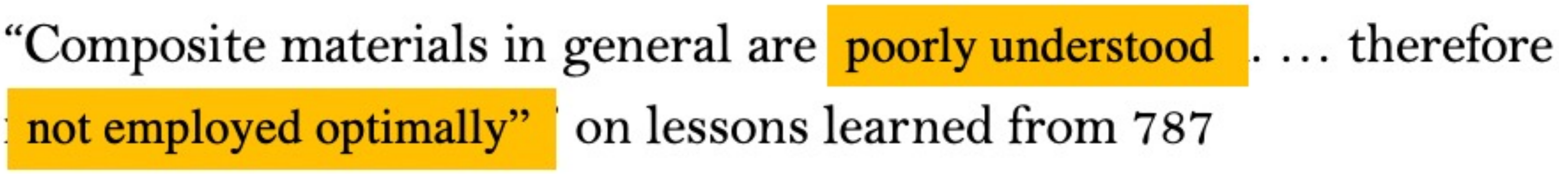
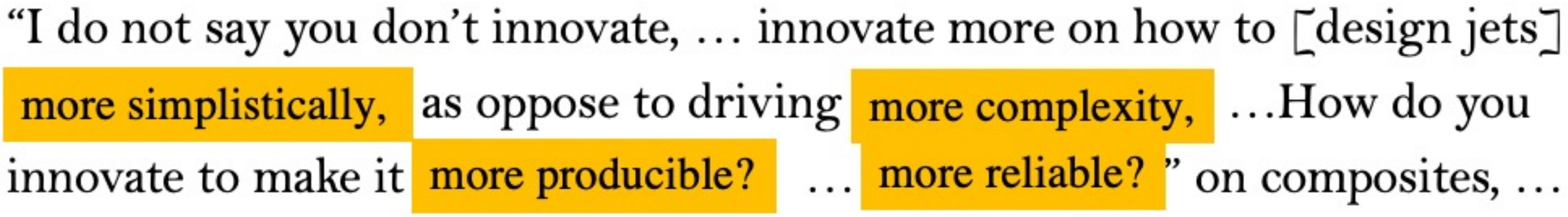
Critique of Composite Materials


“Composite materials in general are poorly understood. … therefore not employed optimally” on lessons learned from 787

By John Byrne, Head, Materials and Structures, Boeing Commercial
at Carbon Fiber in December 2014


“I do not say you don’t innovate, … innovate more on how to [design jets] more simplistically, as oppose to driving more complexity, …How do you innovate to make it more producible? … more reliable?” on composites, …

By Ray Conner, Head, Boeing Commercial, with *WSJ*, April 2015

Better understood and more simplistic

Keys to eliminate self-inflicted complexities

Aluminum	**The new CFRP**
Isotropy	Master stiffness: $A_{11} + A_{22} + 2A_{66}$ = Trace (one and only)
	Scalar product: $F_{ij}\sigma_i\sigma_j + F_i\sigma_i = 1$ (e.g., Tsai-Wu)
	Master failure criterion: omni envelopes (X and X’ only)
Homogeniety	Conditions: [A*] = [D*], [B*] = 0 (less complexity)
	Double-double: $[\pm\Phi/\pm\Psi]_{rT}$ to replace $[0_p/\pm45_q/90_r]_S$
	Rating and scaling: instant answer without recalculation
Constant thickness	Additive manufacturing: tapered to save weight

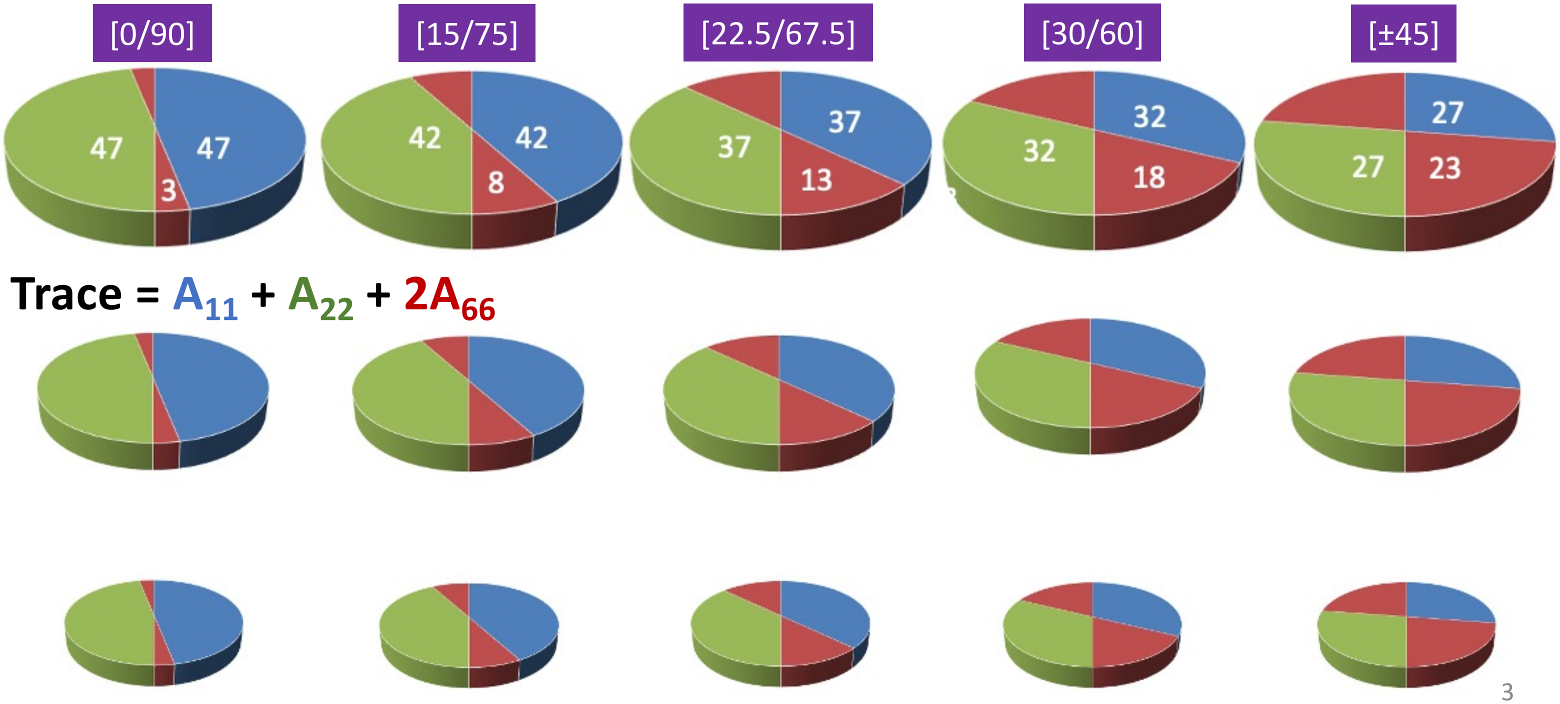
Trace: one master stiffness constant for all laminates

Material

Ply material	Trace
IM6/epoxy	232
IM7/977-3	218
T300/5208	206
IM7/MTM45	195
T800/Cytec	183
IM7/8552	180
T800S/3900	168
T300/F934	168
T700 C-Ply 64	163
AS4/H3501	162
T650/epoxy	160
T4708/MR60H	158
T700/2510	144
AS4/MTM45	143
T700 C-Ply 55	139

Geometry

Figures are the percent particitioning of trace to stiffness components



Trace = $A_{11} + A_{22} + 2A_{66}$

Elastic constants of DD as percentages of trace

Material

Ply material	Trace
IM6/epoxy	232
IM7/977-3	218
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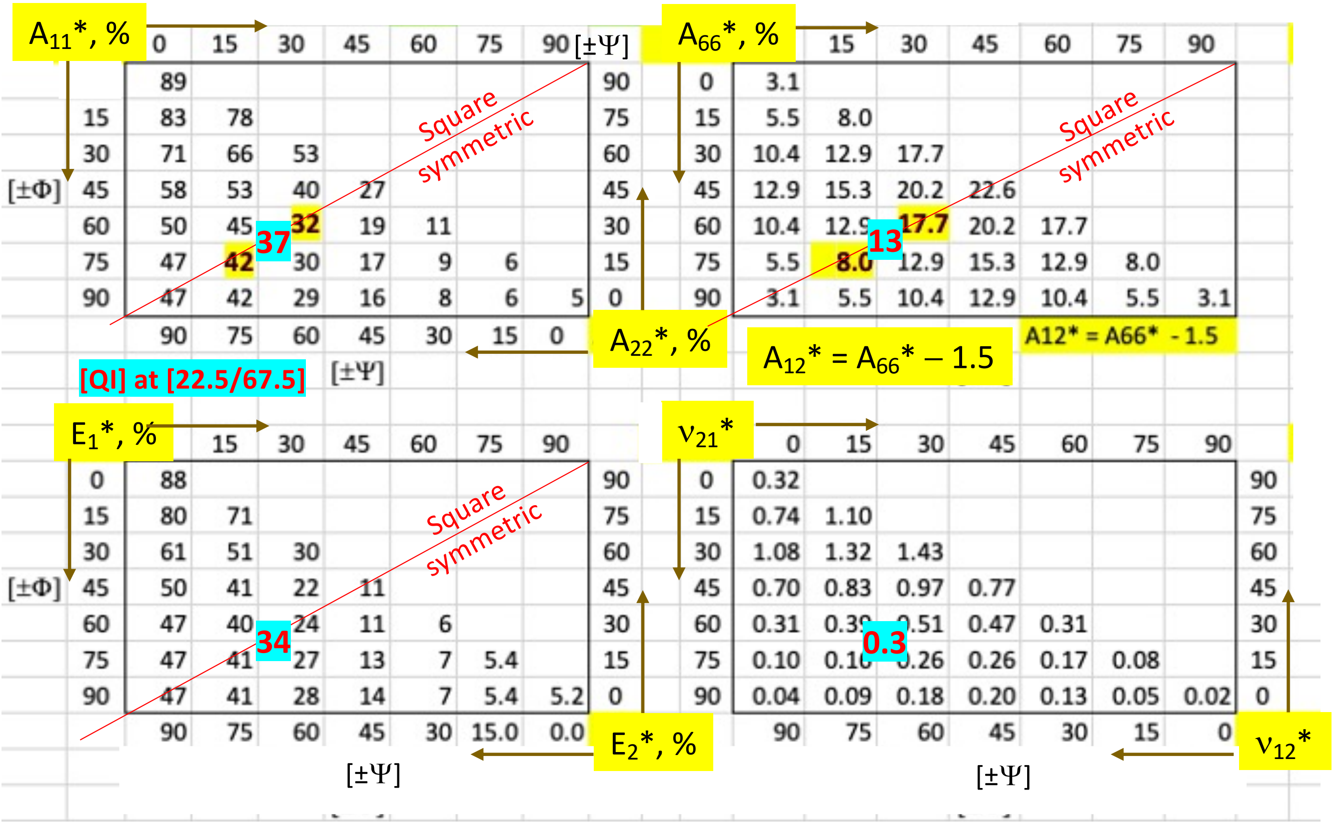
Geometry



		0	15	30	45	60	75	90	[±Ψ]
		89							90
	15	83	78						75
	30	71	66	53					60
[±Φ]	45	58	53	40	27				45
	60	50	45	32	19	11			30
	75	47	42	30	17	9	6		15
	90	47	42	29	16	8	6	5	0
		90	75	60	45	30	15	0	
					[±Ψ]				

		15	30	45	60	75	90
0	3.1						
15	5.5	8.0					
30	10.4	12.9	17.7				
45	12.9	15.3	20.2	22.6			
60	10.4	12.9	17.7	20.2	17.7		
75	5.5	8.0	12.9	15.3	12.9	8.0	
90	3.1	5.5	10.4	12.9	10.4	5.5	3.1
					A12* = A66* - 1.5		

			15	30	45	60	75	90	
	0	88							90
	15	80	71						75
	30	61	51	30					60
[±Φ]	45	50	41	22	11				45
	60	47	40	24	11	6			30
	75	47	41	27	13	7	5.4		15
	90	47	41	28	14	7	5.4	5.2	0
		90	75	60	45	30	15.0	0.0	
					[±Ψ]				

	0	15	30	45	60	75	90	
0	0.32							90
15	0.74	1.10						75
30	1.08	1.32	1.43					60
45	0.70	0.83	0.97	0.77				45
60	0.31	0.39	0.51	0.47	0.31			30
75	0.10	0.10	0.26	0.26	0.17	0.08		15
90	0.04	0.09	0.18	0.20	0.13	0.05	0.02	0
	90	75	60	45	30	15	0	
				[±Ψ]				

Better understood and more simplistic

Keys to eliminate self-inflicted complexities

Aluminum	**The new CFRP**
Isotropy	Master stiffness: $A_{11} + A_{22} + 2A_{66}$ = Trace (one and only) Scalar product: $F_{ij}\sigma_i\sigma_j + F_i\sigma_i = 1$ (e.g., Tsai-Wu) Master failure criterion: omni envelopes (X and X’ only)
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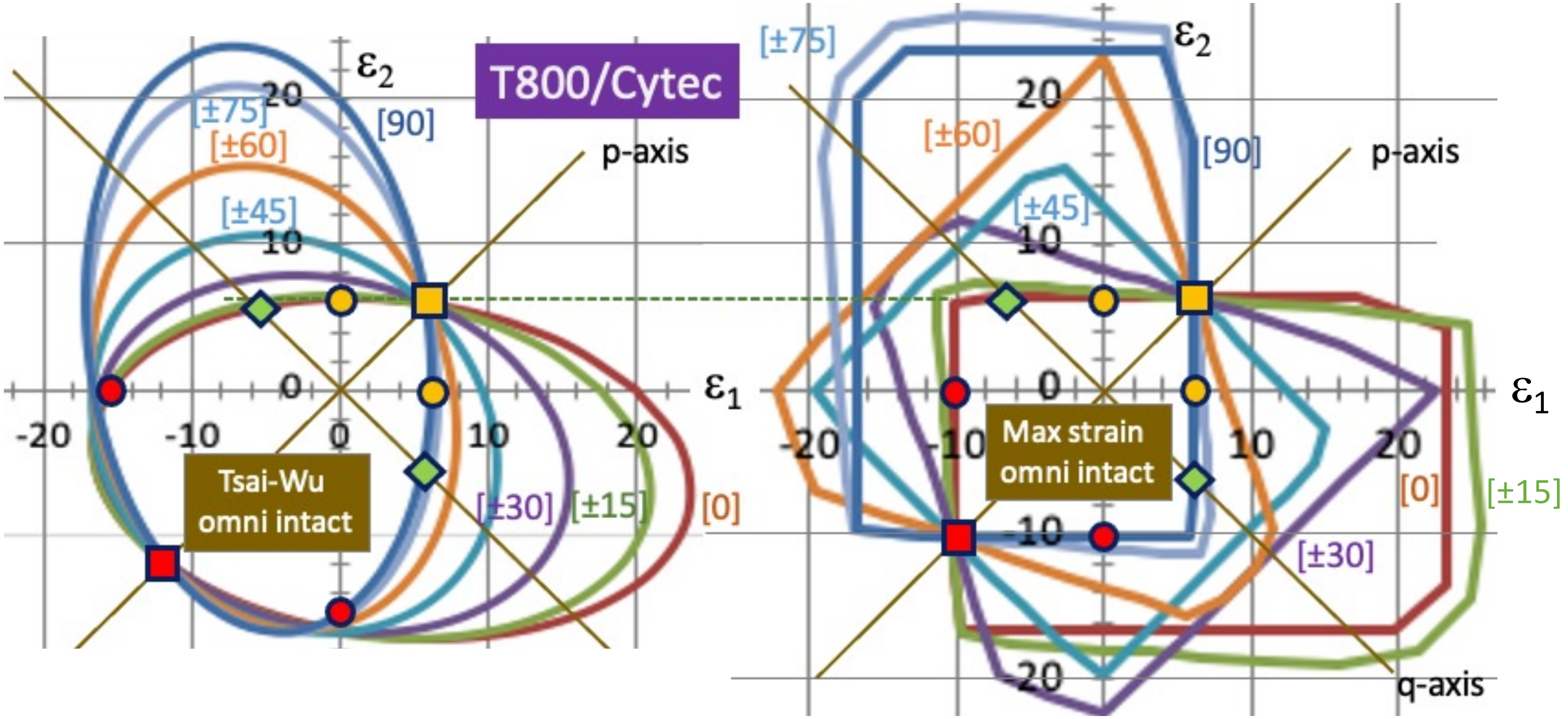
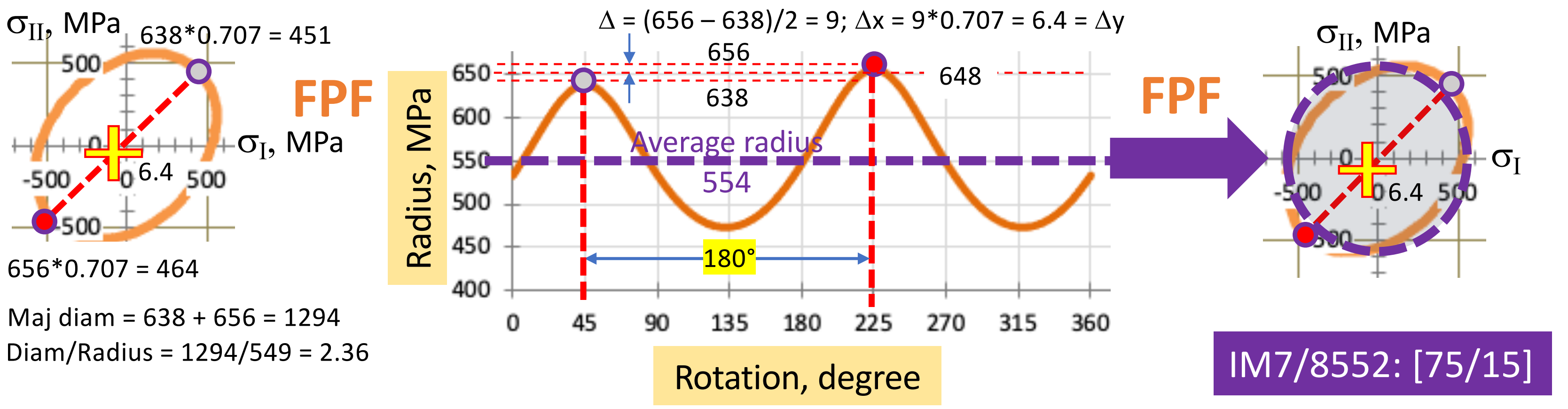
Constant thickness	Additive manufacturing: tapered to save weight

FPF omni envelopes for Tsai-Wu and max strain



FPF/LPF radia, diameters, and center locations



638*0.707 = 451

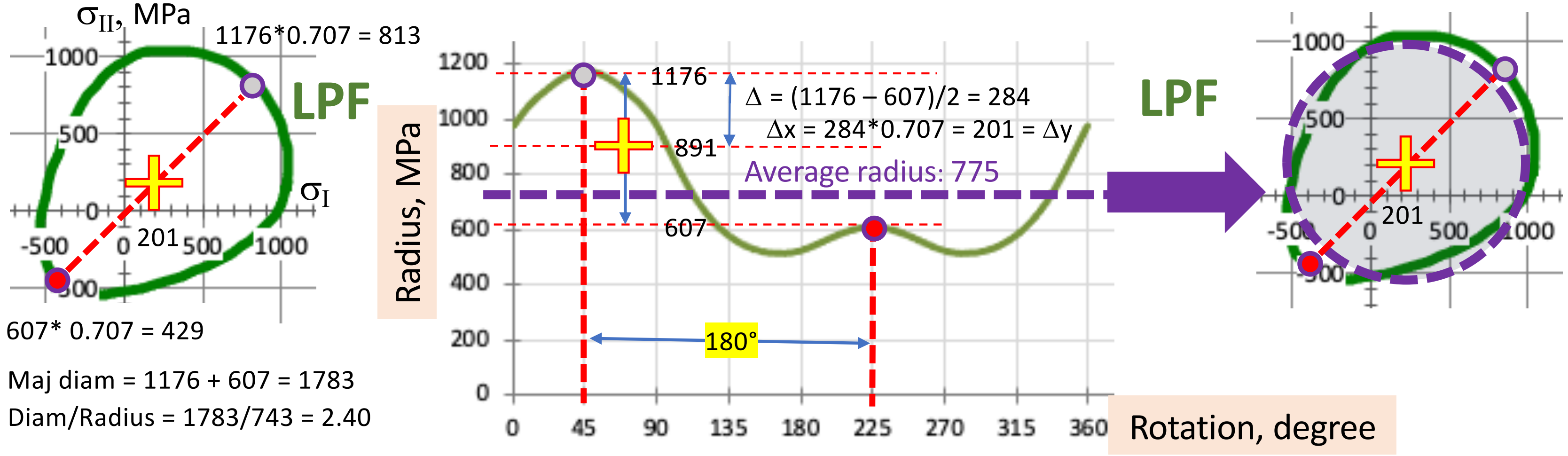
Δ = (656 – 638)/2 = 9; Δx = 9*0.707 = 6.4 = Δy

656*0.707 = 464

Maj diam = 638 + 656 = 1294

Diam/Radius = 1294/549 = 2.36

IM7/8552: [75/15]



1176*0.707 = 813

Δ = (1176 – 607)/2 = 284

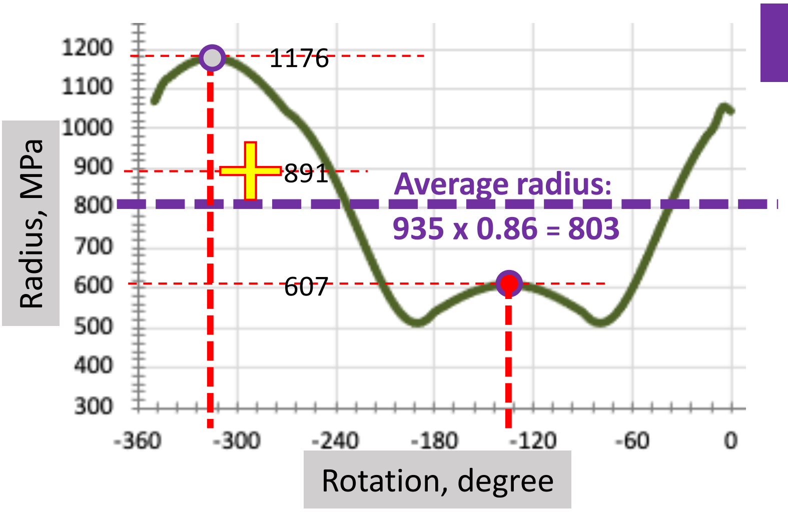
Δx = 284*0.707 = 201 = Δy

607* 0.707 = 429

Maj diam = 1176 + 607 = 1783

Diam/Radius = 1783/743 = 2.40

Absolute and normalized radius of FPF envelope



Master Radius [±Ψ] (columns), [±Φ] (rows)

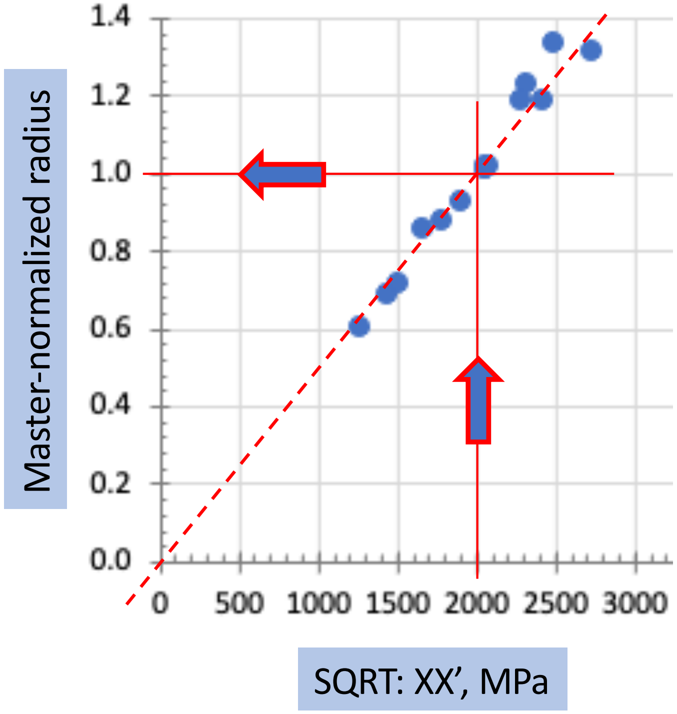
Master Radius	0	15	30	45	60	75	90
0	1332						
15	1249	1169					
30	1065	994	849				
45	915	852	739	690			
60	893	847	739	739	849		
75	996	935	835	852	994	1169	
90	1056	996	893	915	1065	1249	1332

Square symmetric

IM7/8552: [75/15]

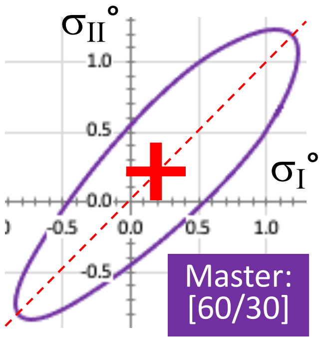
CFRP	Radius*	X	X'	SR XX'
T3/F93	0.61	1314	1220	1266
AS4/H3501	0.69	1447	1447	1447
T3/N52	0.72	1500	1500	1500
IM7/8552	0.86	2326	1200	1671
T7/2510	0.88	2172	1450	1775
T650/ep	0.93	2194	1653	1904
IM7/MTM	1.01	2500	1700	2062
C-Ply 55	1.01	2530	1669	2055
IM7/8552'	1.02	2501	1700	2062
T4708/MR	1.02	2524	1700	2071
IM7/977	1.19	3250	1600	2280
C-Ply 64	1.19	2944	1983	2416
IM6/ep	1.23	3500	1540	2322
T8S/3900	1.32	3000	2500	2739
T800/Cyt	1.34	3768	1656	2498

Material adjustment

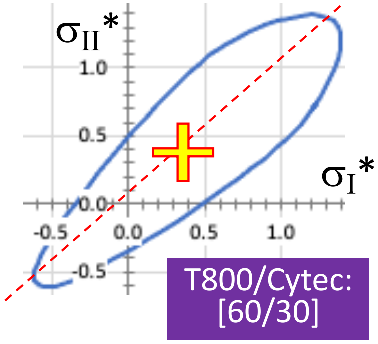


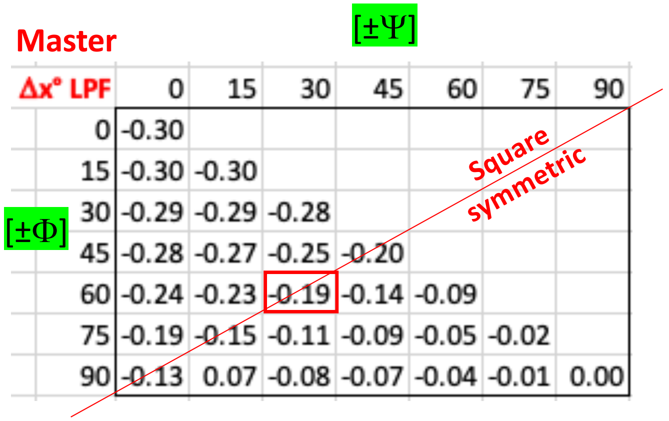
Absolute and normalized envelope center locations

$\Delta x° = \mathbf{-0.19} = \Delta y°$



$\Delta x^* = 2(-0.19) = \mathbf{-0.38} = \Delta y^*$





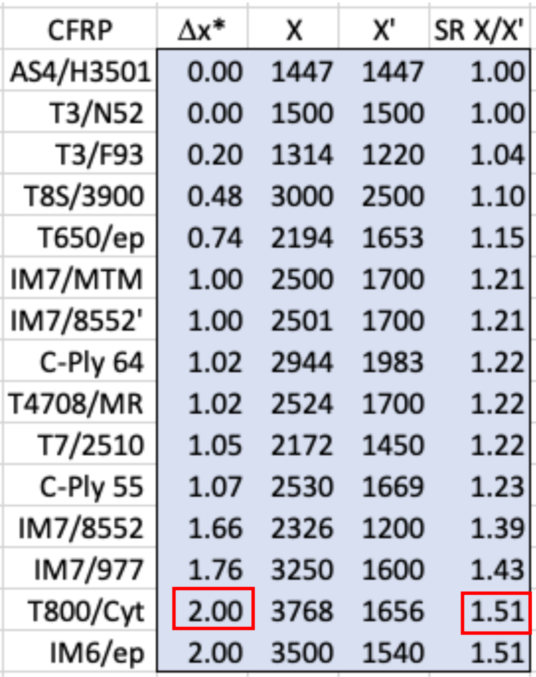
Master

[±Ψ]

[±Φ]

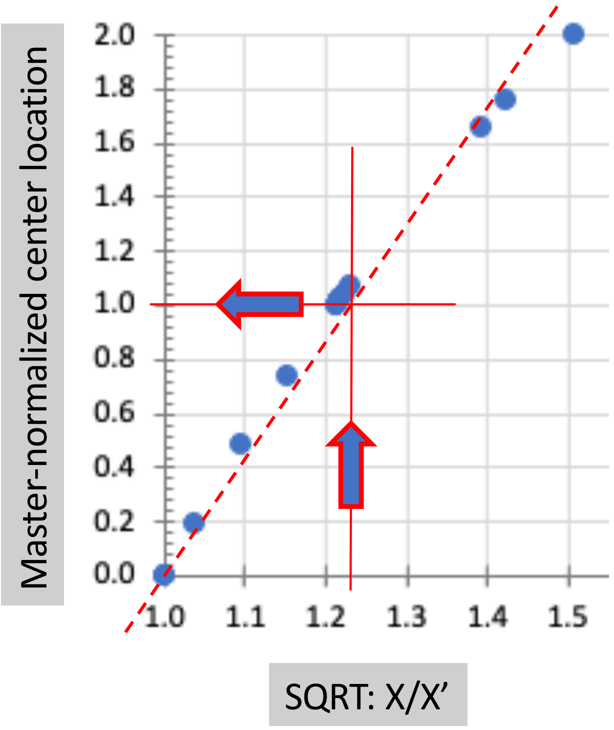
Δx° LPF	0	15	30	45	60	75	90
0	-0.30						
15	-0.30	-0.30					
30	-0.29	-0.29	-0.28				
45	-0.28	-0.27	-0.25	-0.20			
60	-0.24	-0.23	-0.19	-0.14	-0.09		
75	-0.19	-0.15	-0.11	-0.09	-0.05	-0.02	
90	-0.13	0.07	-0.08	-0.07	-0.04	-0.01	0.00

Square symmetric



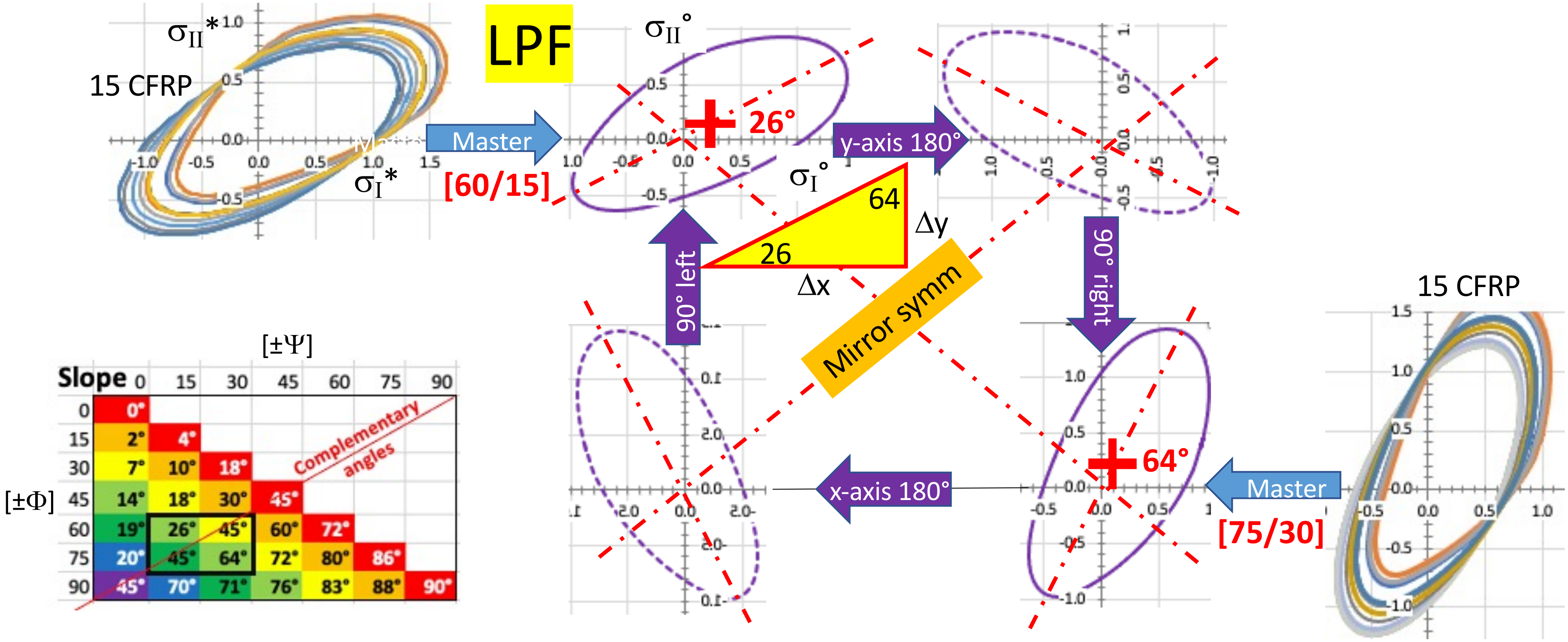
CFRP	Δx*	X	X'	SR X/X'
AS4/H3501	0.00	1447	1447	1.00
T3/N52	0.00	1500	1500	1.00
T3/F93	0.20	1314	1220	1.04
T8S/3900	0.48	3000	2500	1.10
T650/ep	0.74	2194	1653	1.15
IM7/MTM	1.00	2500	1700	1.21
IM7/8552'	1.00	2501	1700	1.21
C-Ply 64	1.02	2944	1983	1.22
T4708/MR	1.02	2524	1700	1.22
T7/2510	1.05	2172	1450	1.22
C-Ply 55	1.07	2530	1669	1.23
IM7/8552	1.66	2326	1200	1.39
IM7/977	1.76	3250	1600	1.43
T800/Cyt	2.00	3768	1656	1.51
IM6/ep	2.00	3500	1540	1.51

Material adjustment



Double transformation between mirrored envelopes

Between [60/15] and [75/30]

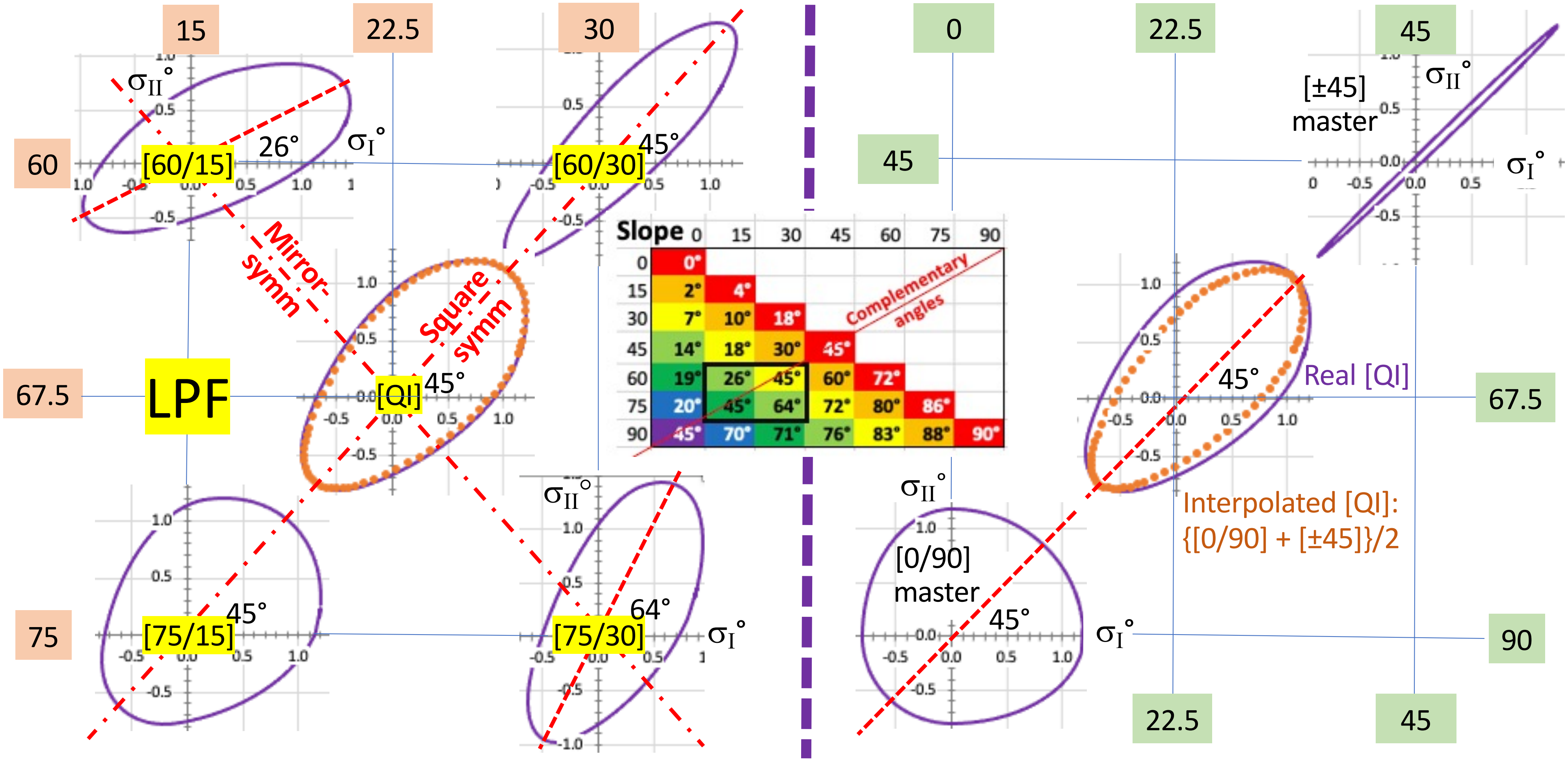


[±Ψ]

Slope [±Φ]	0	15	30	45	60	75	90
0	0°						
15	2°	4°					
30	7°	10°	18°				
45	14°	18°	30°	45°			
60	19°	26°	45°	60°	72°		
75	20°	45°	64°	72°	80°	86°	
90	45°	70°	71°	76°	83°	88°	90°

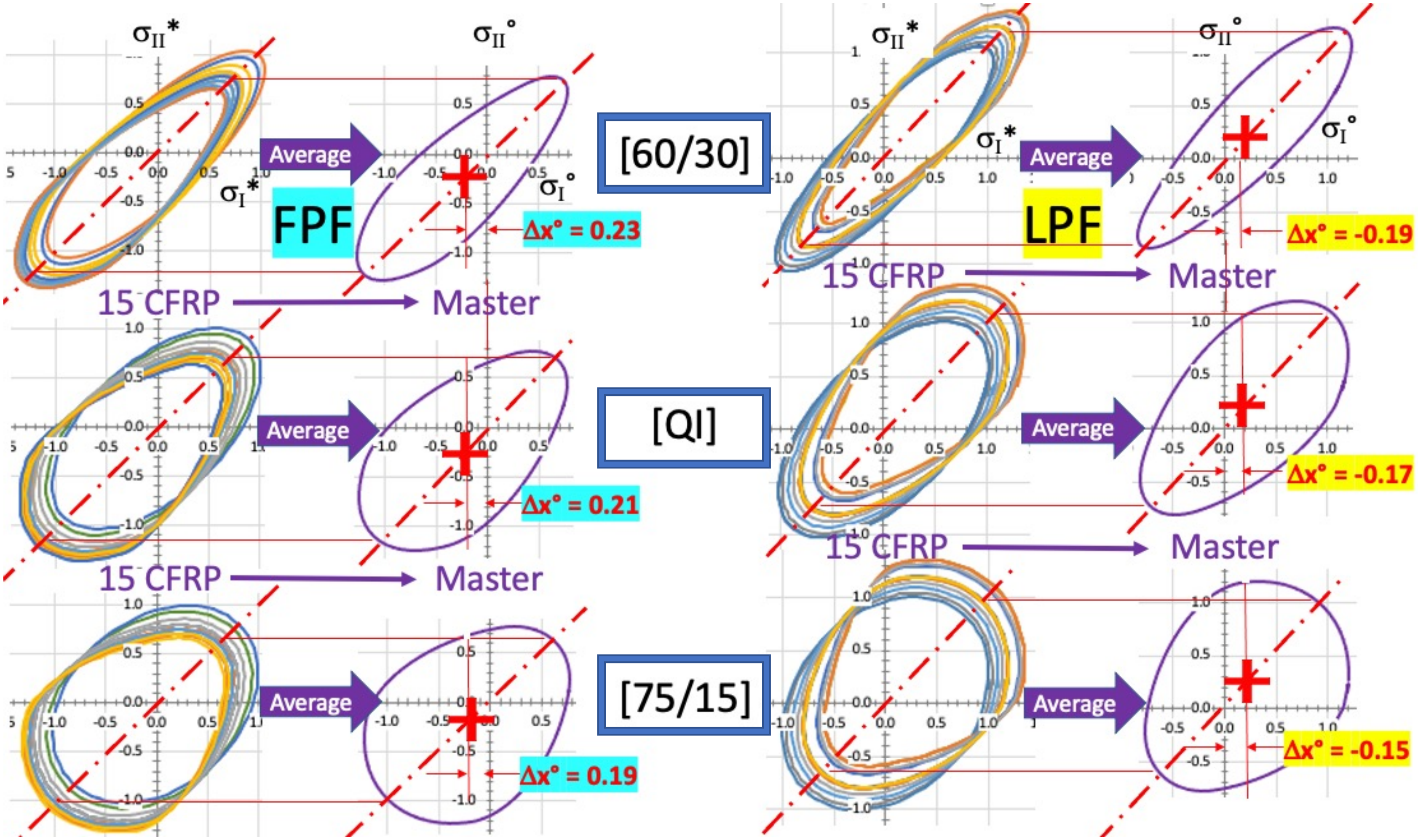
Complementary angles

Interpolation among envelopes

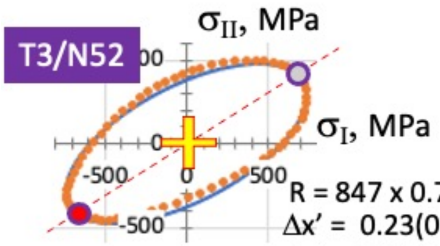


Slope	0	15	30	45	60	75	90
0	0°						
15	2°	4°					
30	7°	10°	18°				
45	14°	18°	30°	45°			
60	19°	26°	45°	60°	72°		
75	20°	45°	64°	72°	80°	86°	
90	45°	70°	71°	76°	83°	88°	90°

Master envelopes from 15 radius-normalized CFRP



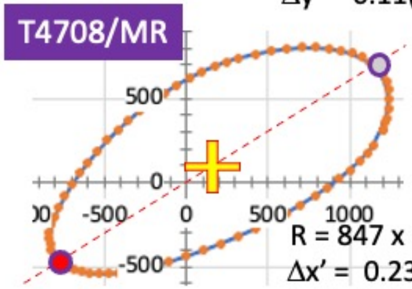
Recovery of individual from master envelope



R = 847 x 0.72 = 610

$\Delta x' = 0.23(0 - 1) = -0.23$

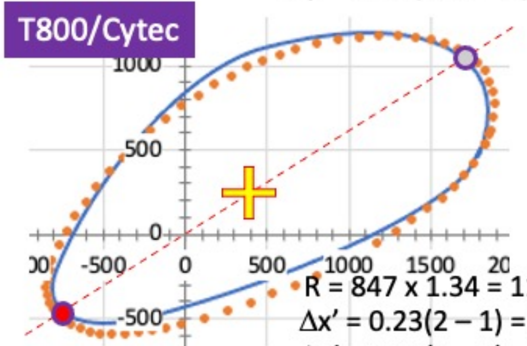
$\Delta y' = 0.11(0 - 1) = -0.11$



R = 847 x 1.02 = 864

$\Delta x' = 0.23(1.02 - 1) = 0.00$

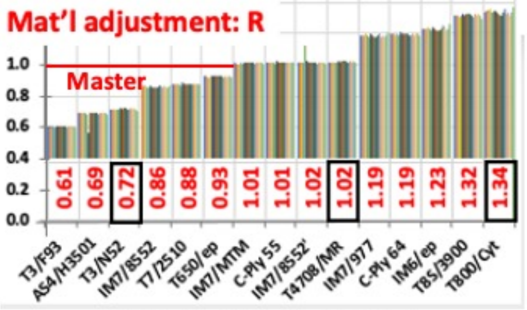
$\Delta y' = 0.11(1.02 - 1) = 0.00$



R = 847 x 1.34 = 1135

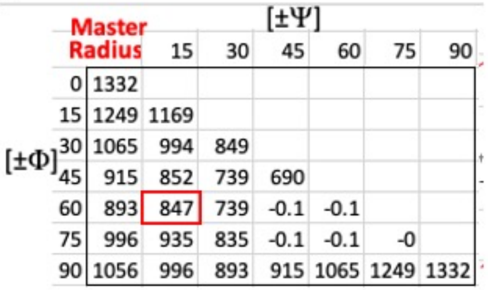
$\Delta x' = 0.23(2 - 1) = 0.23$ (= 0.23 x 847 = 195)

$\Delta y' = 0.11(2 - 1) = 0.11$ (= 0.11 x 847 = 93)

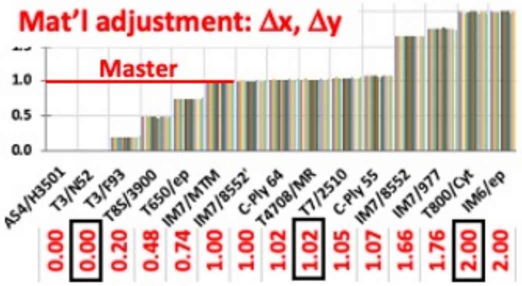


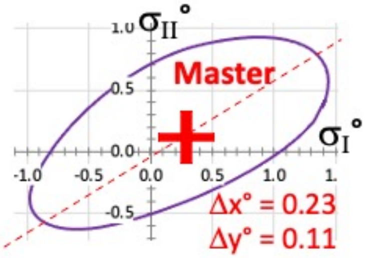
[60/15] LPF

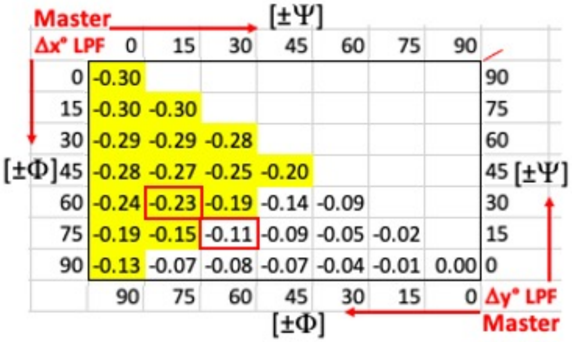
$$\sigma_I = R\sigma_I^* = R(\sigma_I^\circ - \Delta x')$$
$$\sigma_{II} = R\sigma_{II}^* = R(\sigma_{II}^\circ - \Delta y')$$



		[±Ψ]					
[±Φ]	Master Radius	15	30	45	60	75	90
0	1332						
15	1249	1169					
30	1065	994	849				
45	915	852	739	690			
60	893	847	739	-0.1	-0.1		
75	996	935	835	-0.1	-0.1	-0	
90	1056	996	893	915	1065	1249	1332

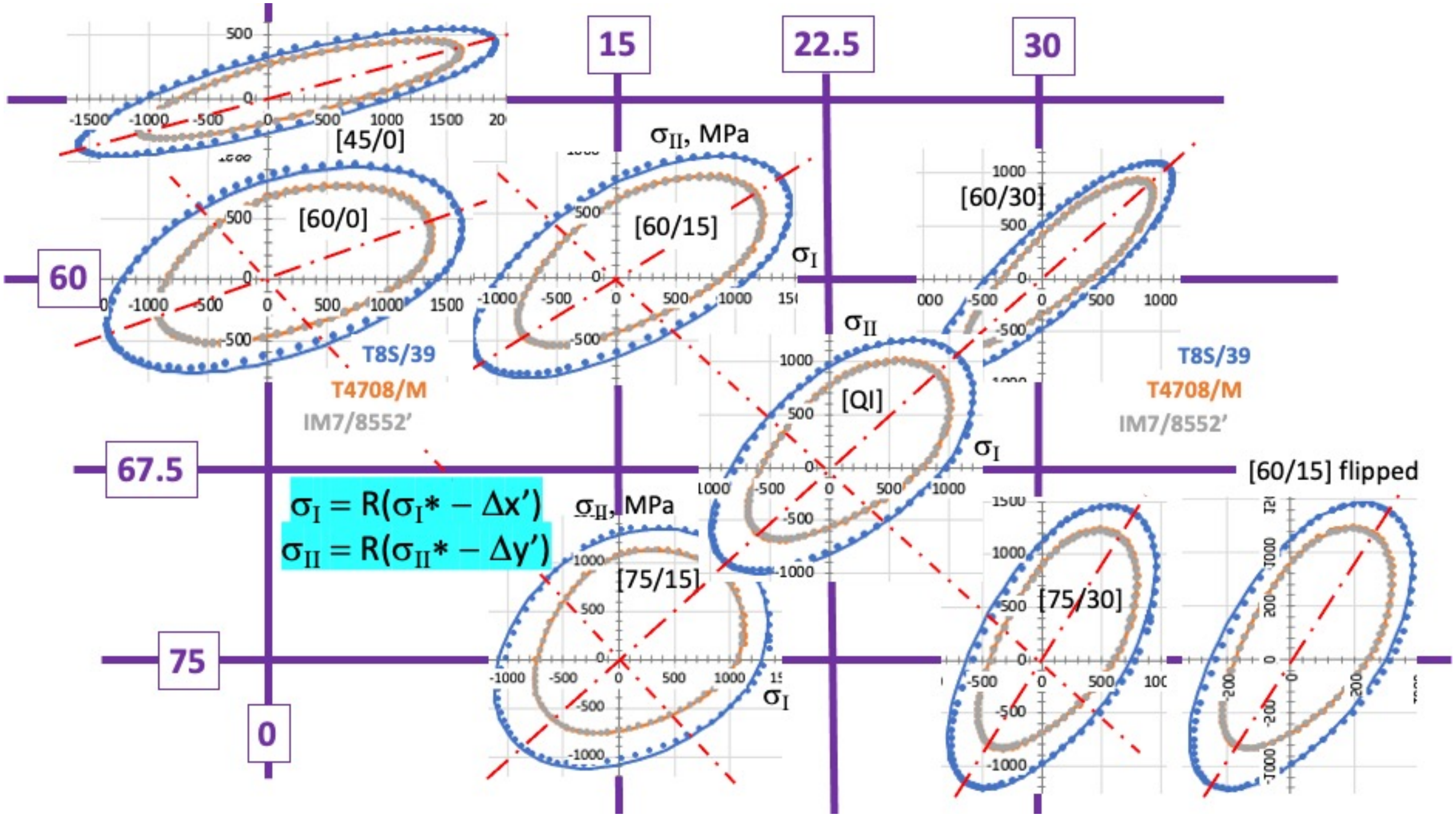






Master				[±Ψ]				
Δx° LPF	0	15	30	45	60	75	90	
0	-0.30							90
15	-0.30	-0.30						75
30	-0.29	-0.29	-0.28					60
[±Φ] 45	-0.28	-0.27	-0.25	-0.20				45 [±Ψ]
60	-0.24	-0.23	-0.19	-0.14	-0.09			30
75	-0.19	-0.15	-0.11	-0.09	-0.05	-0.02		15
90	-0.13	-0.07	-0.08	-0.07	-0.04	-0.01	0.00	0
	90	75	60	45	30	15	0	Δy° LPF
				[±Φ]				Master

DD failure envelopes: recovered from master



$$\sigma_I = R(\sigma_I* - \Delta x')$$
$$\sigma_{II} = R(\sigma_{II}* - \Delta y')$$

Better understood and more simplistic

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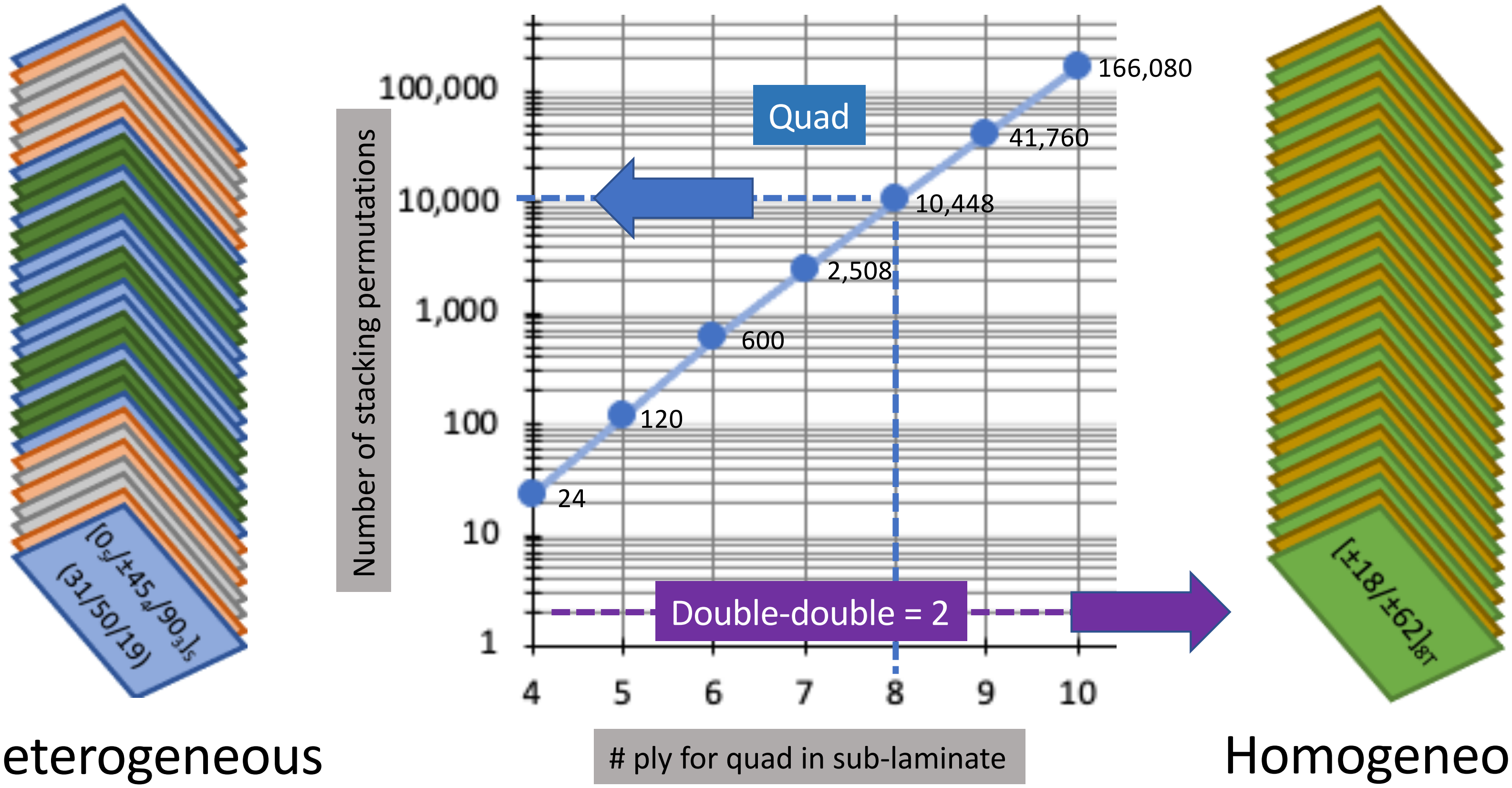
Stacking sequence permutations: quad vs DD

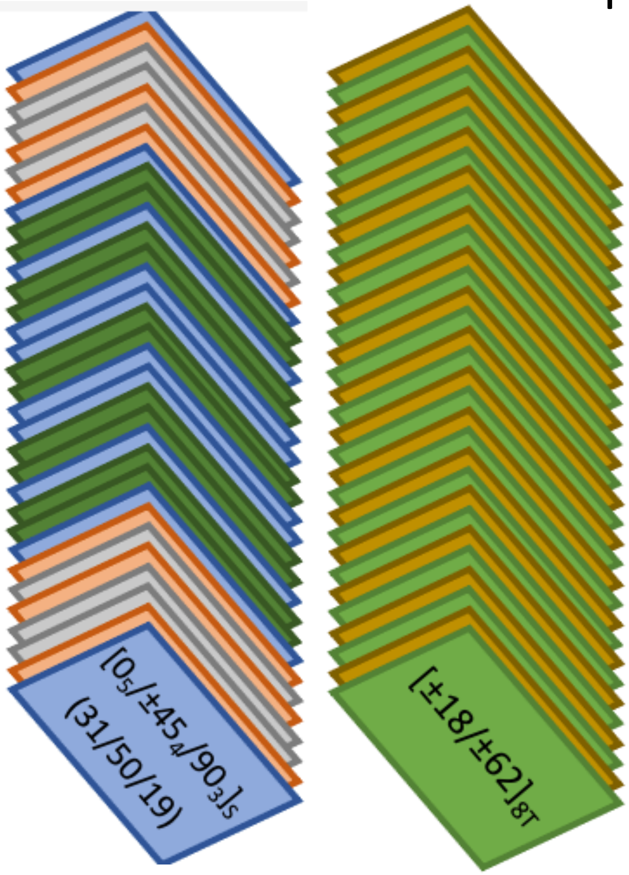


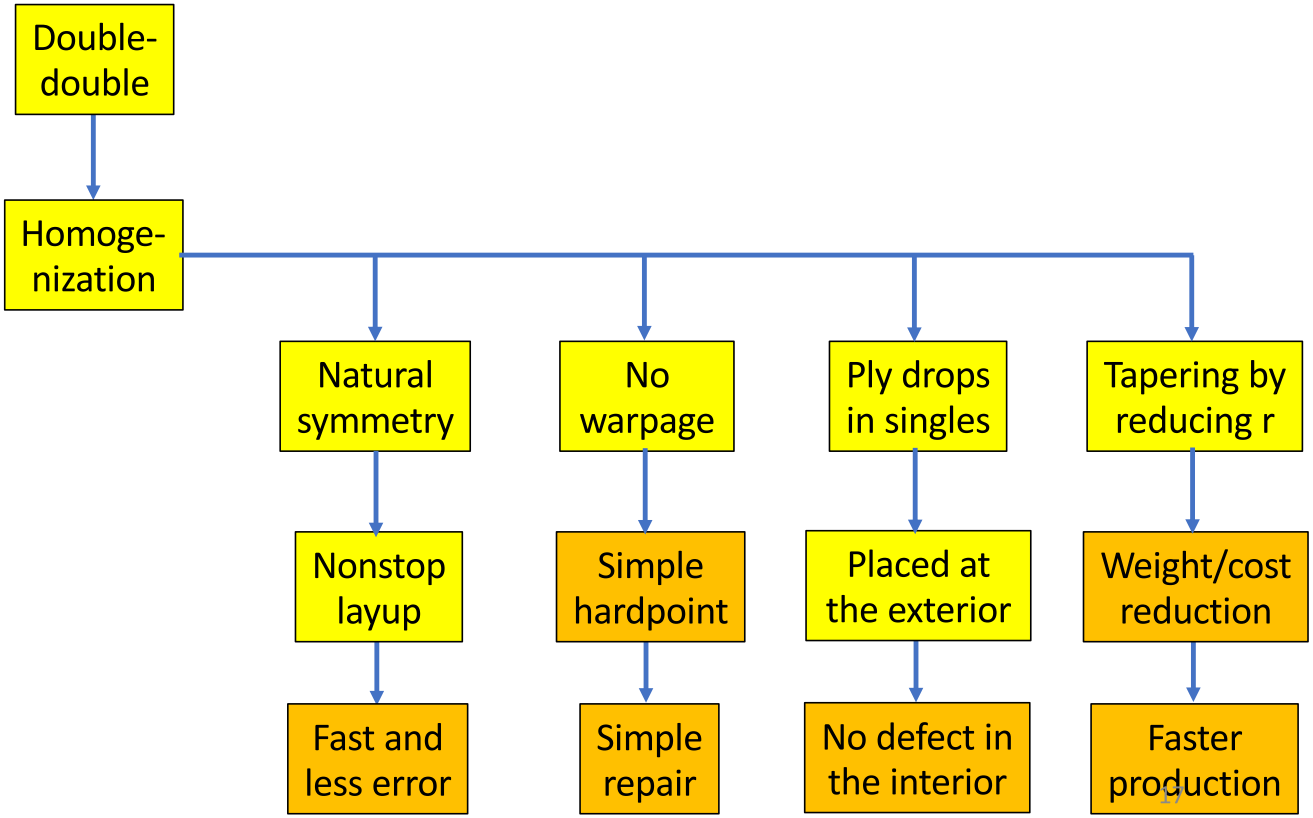
Heterogeneous

Homogeneous

DD opportunities in manufacturing

Not possible with quad





Double-double

Homogenization

Natural symmetry

Nonstop layup

Fast and less error

No warpage

Simple hardpoint

Simple repair

Ply drops in singles

Placed at the exterior

No defect in the interior

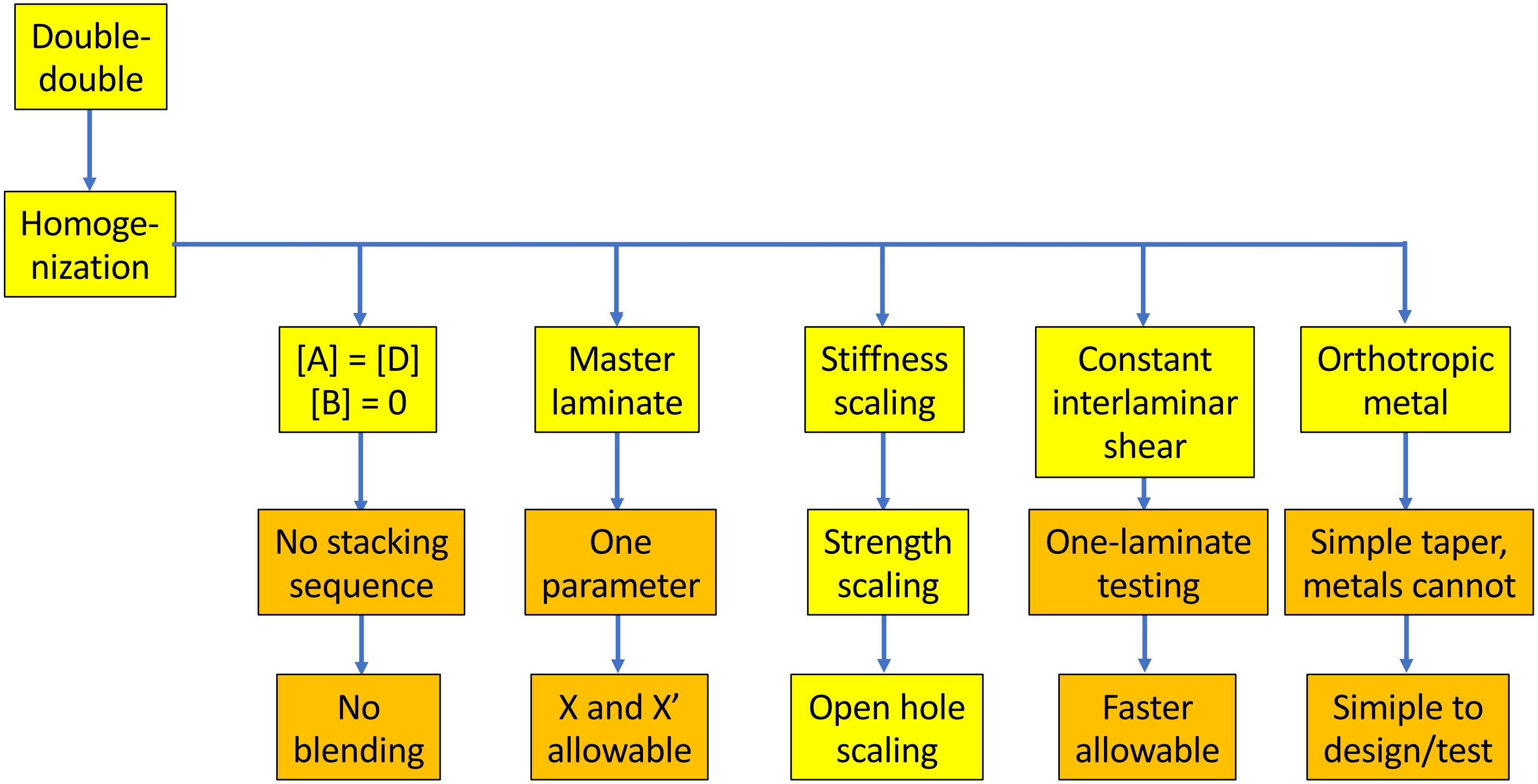
Tapering by reducing r

Weight/cost reduction

Faster production

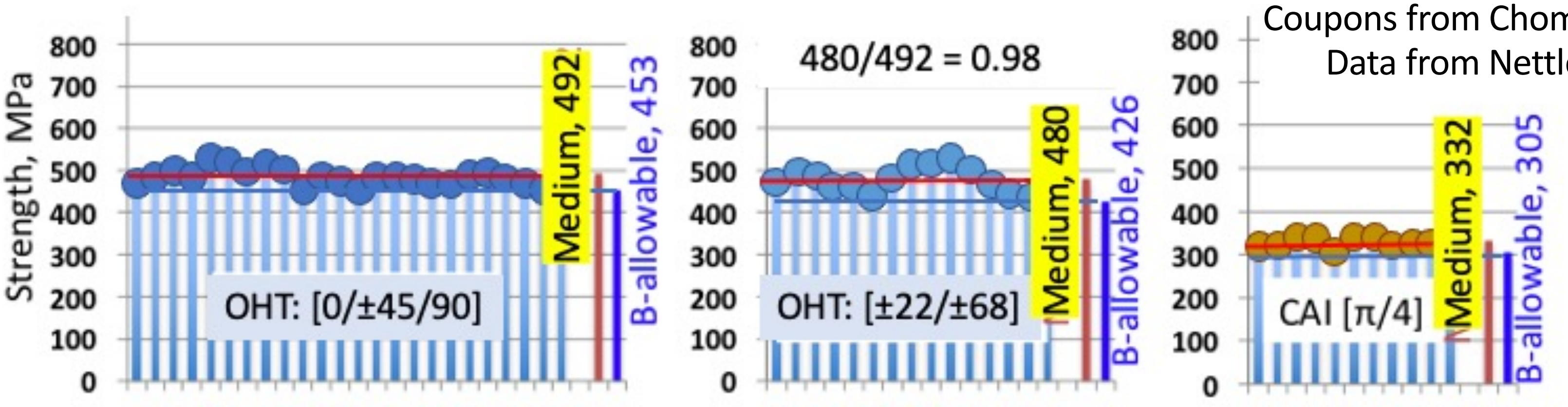
DD opportunities in design

Not possible with quad

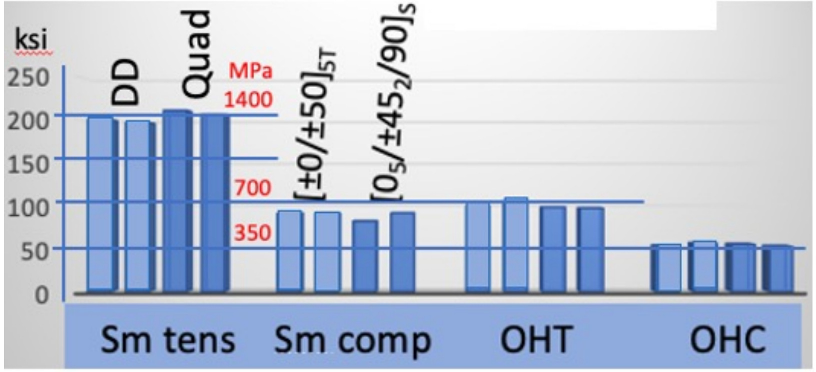


- Double-double
  - Homogenization
    - [A] = [D]
[B] = 0
      - No stacking sequence
        - No blending
    - Master laminate
      - One parameter
        - X and X’ allowable
    - Stiffness scaling
      - Strength scaling
        - Open hole scaling
    - Constant interlaminar shear
      - One-laminate testing
        - Faster allowable
    - Orthotropic metal
      - Simple taper, metals cannot
        - Simiple to design/test

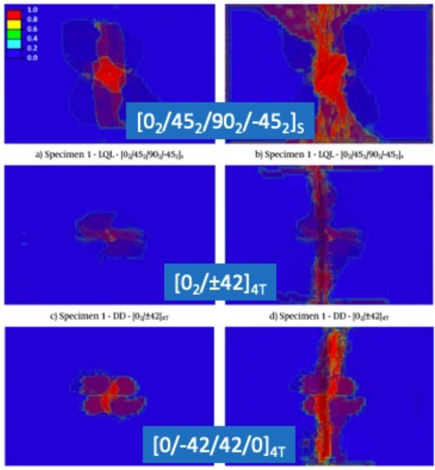
OHT and CAI data for quad and DD

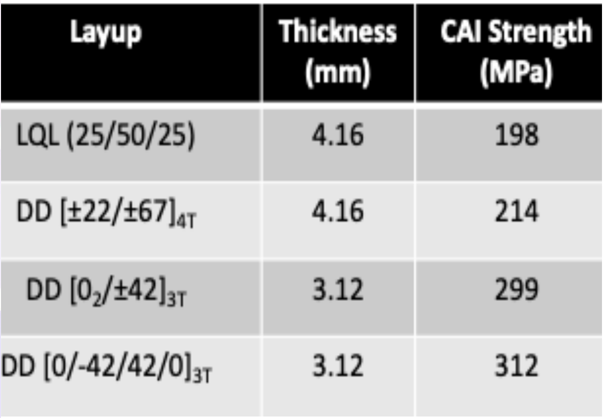


Coupons from Chomarat/Hexcell
Data from Nettles, NASA



Coupons from Toray
Data from Waruna, NIAR



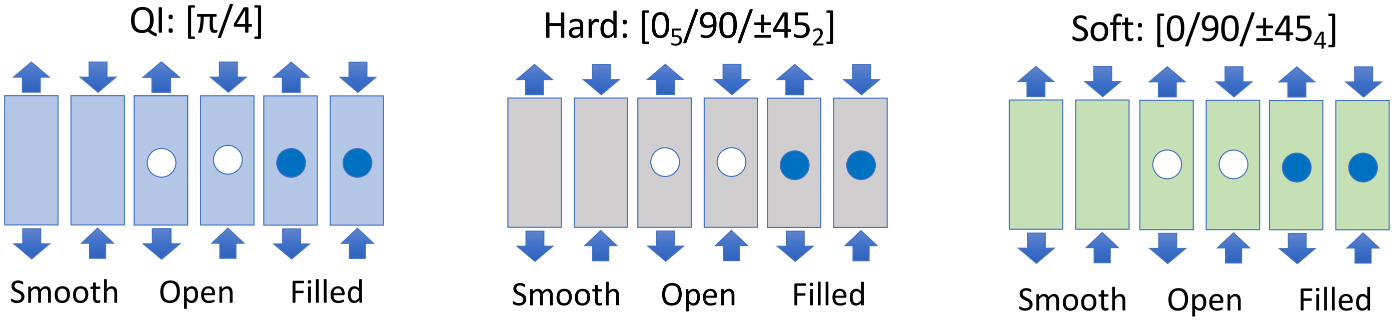


Layup	Thickness (mm)	CAI Strength (MPa)
LQL (25/50/25)	4.16	198
DD $[\pm 22/\pm 67]_{4T}$	4.16	214
DD $[0_2/\pm 42]_{3T}$	3.12	299
DD $[0/-42/42/0]_{3T}$	3.12	312

Data from B. Falzon et al, Queen’s Univ Belfast, and RMIT

Design allowable generation: [0/90] only + as-built

Legacy



QI: [π/4]

Hard: $[0_5/90/\pm45_2]$

Soft: $[0/90/\pm45_4]$

Smooth Open Filled

Smooth Open Filled

Smooth Open Filled

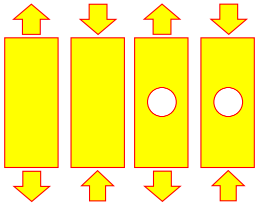
Proposed

Reduction

18/4 = 4.5

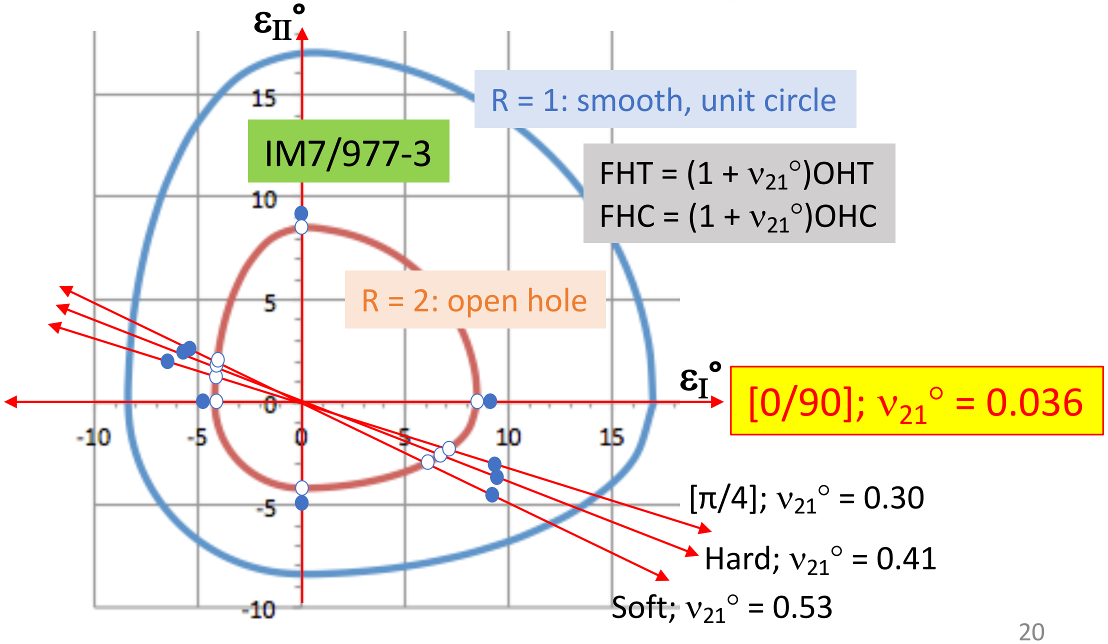
4/18 = 22%

Target laminate



Smooth Notched

E_x°, X, X', OHT, OHC



R = 1: smooth, unit circle

IM7/977-3

FHT = $(1 + \nu_{21}°)$OHT

FHC = $(1 + \nu_{21}°)$OHC

R = 2: open hole

[0/90]; $\nu_{21}° = 0.036$

[π/4]; $\nu_{21}° = 0.30$

Hard; $\nu_{21}° = 0.41$

Soft; $\nu_{21}° = 0.53$

Better understood and more simplistic

Keys to eliminate self-inflicted complexities

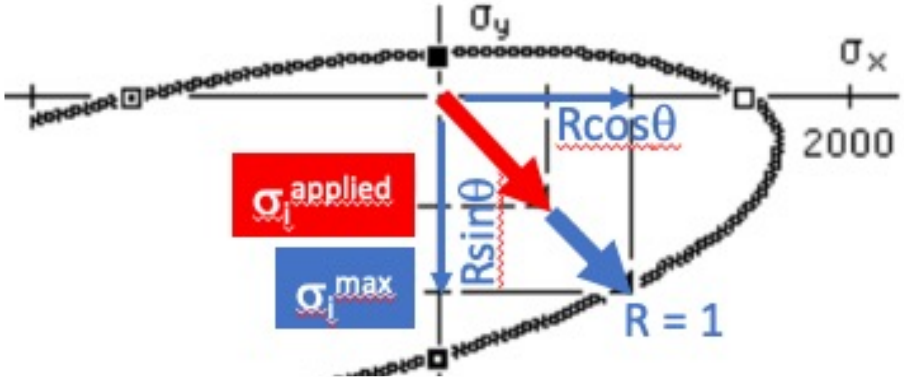
Aluminum	**The new CFRP**
Isotropy	Master stiffness: $A_{11} + A_{22} + 2A_{66}$ = Trace (one and only) Scalar product: $F_{ij}\sigma_i\sigma_j + F_i\sigma_i = 1$ (e.g., Tsai-Wu) Master failure criterion: omni envelopes (X and X’ only)
Homogeniety	Conditions: $[A^*] = [D^*]$, $[B^*] = 0$ (less complexity) Double-double: $[\pm\Phi/\pm\Psi]_{rT}$ to replace $[0_p/\pm45_q/90_r]_S$ Rating and scaling: instant answer without recalculation
Constant thickness	Additive manufacturing: tapered to save weight

A key improvement over failure criterion

Strength ratio R

$\sigma_i^{max} = R\sigma_i^{applied}$, or

$\epsilon_i^{max} = R\epsilon_i^{applied}$



1) When R = 1, failure occurs
2) When R = 2, stress can be doubled or thickness reduced to 1/2 before failure
3) When R = 0.5, stress can be 1/2 or thickness doubled before failure
4) When applied stress is unity, the resulting R-value is the strength

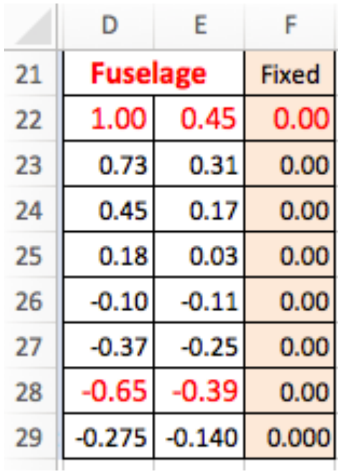
For Tsai-Wu failure $\mathbf{F_{ij}\sigma_i\sigma_j + F_i\sigma_i = 1}$;

substitute and solve for R:

$$\mathbf{[F_{ij}\sigma_i\sigma_j]R^2 + [F_i\sigma_i]R - 1 = 0}$$

Scaling options in input and output data

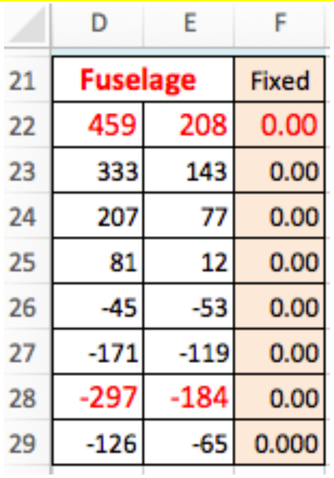
Input: normalized vectors



	D	E	F
21	**Fuselage**		Fixed
22	1.00	0.45	0.00
23	0.73	0.31	0.00
24	0.45	0.17	0.00
25	0.18	0.03	0.00
26	-0.10	-0.11	0.00
27	-0.37	-0.25	0.00
28	-0.65	-0.39	0.00
29	-0.275	-0.140	0.000

IM7/977-3
$[0_2/\pm 75]$

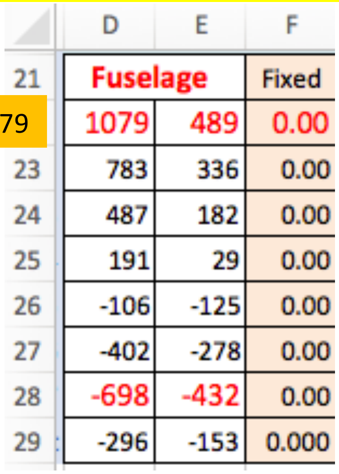
Input: actual stresses



	D	E	F
21	**Fuselage**		Fixed
22	459	208	0.00
23	333	143	0.00
24	207	77	0.00
25	81	12	0.00
26	-45	-53	0.00
27	-171	-119	0.00
28	-297	-184	0.00
29	-126	-65	0.000

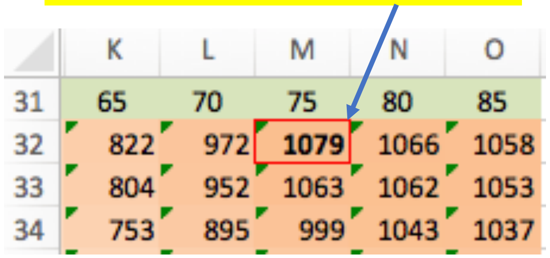
459 x 2.34 = 1079

Input: failure stresses



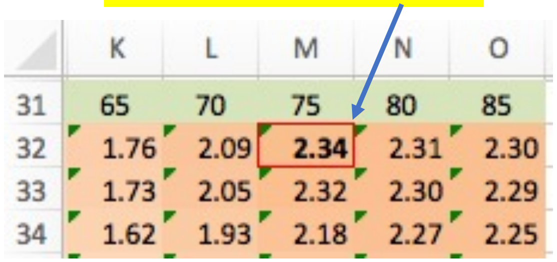
	D	E	F
21	**Fuselage**		Fixed
22	1079	489	0.00
23	783	336	0.00
24	487	182	0.00
25	191	29	0.00
26	-106	-125	0.00
27	-402	-278	0.00
28	-698	-432	0.00
29	-296	-153	0.000

Failure stress: R = 1079 MPa



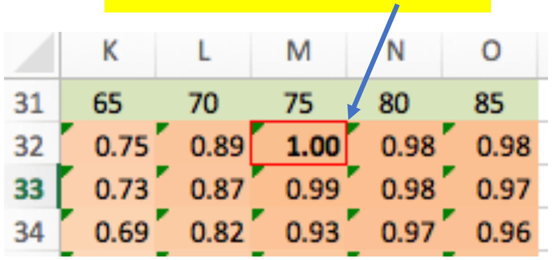
	K	L	M	N	O
31	65	70	75	80	85
32	822	972	**1079**	1066	1058
33	804	952	1063	1062	1053
34	753	895	999	1043	1037

Safety factor: R = 2.34



	K	L	M	N	O
31	65	70	75	80	85
32	1.76	2.09	**2.34**	2.31	2.30
33	1.73	2.05	2.32	2.30	2.29
34	1.62	1.93	2.18	2.27	2.25

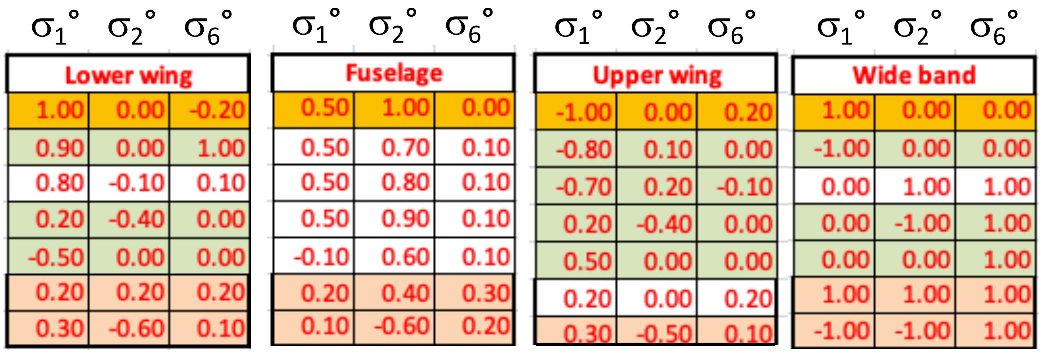
Failure stress: R = 1.00



	K	L	M	N	O
31	65	70	75	80	85
32	0.75	0.89	**1.00**	0.98	0.98
33	0.73	0.87	0.99	0.98	0.97
34	0.69	0.82	0.93	0.97	0.96

Working stress of various structural compontents

Multiple component load sets

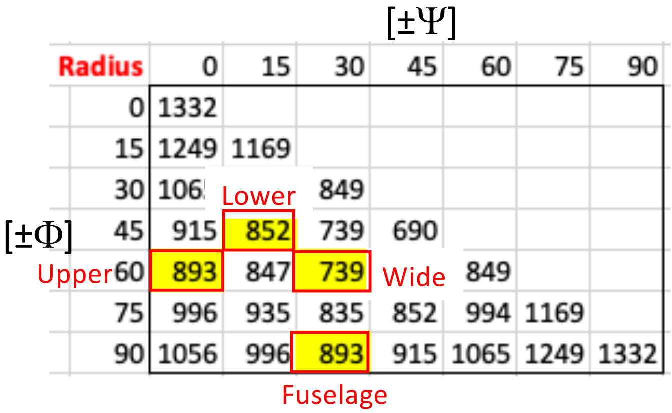


Lower wing		
σ_1°	σ_2°	σ_6°
1.00	0.00	-0.20
0.90	0.00	1.00
0.80	-0.10	0.10
0.20	-0.40	0.00
-0.50	0.00	0.00
0.20	0.20	0.20
0.30	-0.60	0.10

Fuselage		
σ_1°	σ_2°	σ_6°
0.50	1.00	0.00
0.50	0.70	0.10
0.50	0.80	0.10
0.50	0.90	0.10
-0.10	0.60	0.10
0.20	0.40	0.30
0.10	-0.60	0.20

Upper wing		
σ_1°	σ_2°	σ_6°
-1.00	0.00	0.20
-0.80	0.10	0.00
-0.70	0.20	-0.10
0.20	-0.40	0.00
0.50	0.00	0.00
0.20	0.00	0.20
0.30	-0.50	0.10

Wide band		
σ_1°	σ_2°	σ_6°
1.00	0.00	0.00
-1.00	0.00	0.00
0.00	1.00	1.00
0.00	-1.00	1.00
0.00	0.00	1.00
1.00	1.00	1.00
-1.00	-1.00	1.00



		[±Ψ]						
	Radius	0	15	30	45	60	75	90
	0	1332						
	15	1249	1169					
	30	106	Lower	849				
[±Φ]	45	915	852	739	690			
Upper	60	893	847	739	Wide	849		
	75	996	935	835	852	994	1169	
	90	1056	996	893	915	1065	1249	1332

Fuselage

Best DD and working stress

		Best DD	[45/15]	[90/30]	[60/0]	[60/30]
Trace	#	CFRP	Lower wing	Fuselage	Upper wing	Wideband
162	1	AS4/H35	212	429	586	167
232	2	IM6/ep	250	537	633	209
168	3	T3/F93	166	353	457	134
206	4	T3/N52	185	371	545	149
139	5	C-Ply 55	252	535	665	207
163	6	C-Ply 64	262	535	752	213
218	7	IM7/977	283	630	673	240
183	8	T800/Cyt	245	534	639	200
180	9	IM7/8552	231	590	516	213
195	10	IM7/MTM	318	754	717	275
144	11	T7/2510	188	393	535	151
160	12	T650/EP	231	482	638	188
181	13	IM7/8552'	274	602	687	227
158	14	T4708/MR	187	385	550	146
168	15	T8S/3900	445	1027	1028	376
		Average	249	544	641	206

Normalized working stress

Stiffness

			[45/15]	[90/30]	[60/0]	[60/30]		
Trace	#	CFRP	Lower	Fuselag	Upper	Wide	Ave	cv
139	5	C-Ply 55	1.01	0.98	1.04	1.00	1.01	2%
144	11	T7/2510	0.76	0.72	0.83	0.73	0.76	7%
158	14	T4708/MR	0.75	0.71	0.86	0.71	0.76	9%
160	12	T650/EP	0.93	0.89	0.99	0.91	0.93	5%
162	1	AS4/H35	0.85	0.79	0.91	0.81	0.84	7%
163	6	C-Ply 64	1.05	0.98	1.17	1.03	1.06	8%
168	3	T3/F93	0.67	0.65	0.71	0.65	0.67	4%
168	15	T8S/3900	1.79	1.89	1.60	1.82	1.78	7%
180	9	IM7/8552	0.93	1.08	0.80	1.03	0.96	13%
181	13	IM7/8552'	1.10	1.11	1.07	1.10	1.10	1%
183	8	T800/Cyt	0.99	0.98	1.00	0.97	0.98	1%
195	10	IM7/MTM	1.28	1.39	1.12	1.33	1.28	9%
206	4	T3/N52	0.74	0.68	0.85	0.72	0.75	10%
218	7	IM7/977	1.14	1.16	1.05	1.16	1.13	5%
232	2	IM6/ep	1.01	0.99	0.99	1.01	1.00	1%

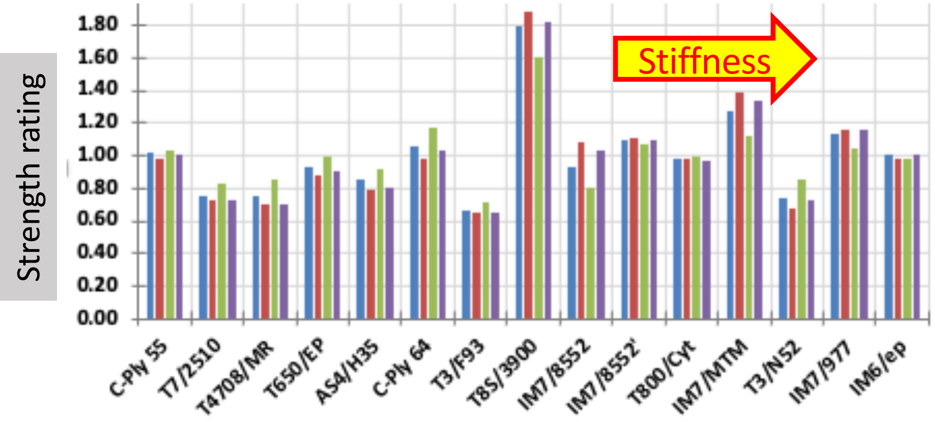
Rating

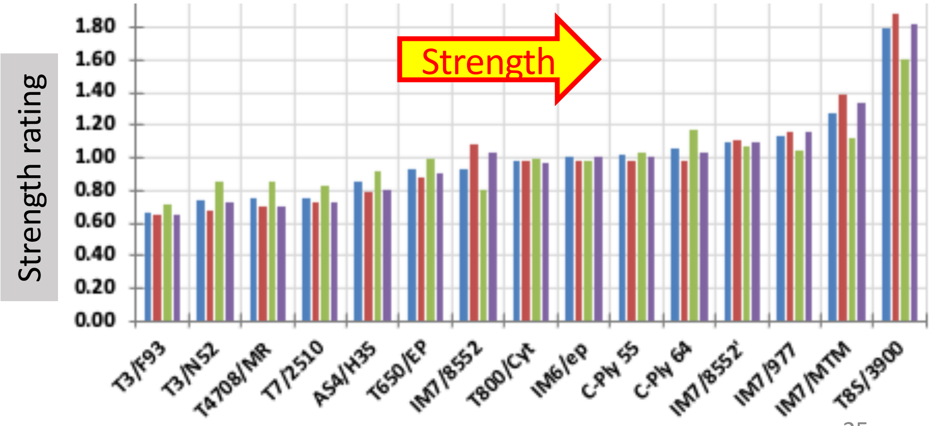
CFRP	Trace	Strength
AS4/H35	162	0.84
IM6/ep	232	1.00
T3/F93	168	0.67
T3/N52	206	0.75
C-Ply 55	139	1.01
C-Ply 64	163	1.06
IM7/977	218	1.13
T800/Cyt	183	0.98
IM7/8552	180	0.96
IM7/MTM	195	1.28
T7/2510	144	0.76
T650/EP	160	0.93
IM7/8552'	181	1.10
T4708/MR	158	0.76
T8S/3900	168	1.78

Normalized working stress

Strength

			[45/15]	[90/30]	[60/0]	[60/30]		
Trace	#	CFRP	Lower	Fuselag	Upper	Wide	Ave	cv
168	3	T3/F93	0.67	0.65	0.71	0.65	0.67	4%
206	4	T3/N52	0.74	0.68	0.85	0.72	0.75	10%
158	14	T4708/MR	0.75	0.71	0.86	0.71	0.76	9%
144	11	T7/2510	0.76	0.72	0.83	0.73	0.76	7%
162	1	AS4/H35	0.85	0.79	0.91	0.81	0.84	7%
160	12	T650/EP	0.93	0.89	0.99	0.91	0.93	5%
180	9	IM7/8552	0.93	1.08	0.80	1.03	0.96	13%
183	8	T800/Cyt	0.99	0.98	1.00	0.97	0.98	1%
232	2	IM6/ep	1.01	0.99	0.99	1.01	1.00	1%
139	5	C-Ply 55	1.01	0.98	1.04	1.00	1.01	2%
163	6	C-Ply 64	1.05	0.98	1.17	1.03	1.06	8%
181	13	IM7/8552'	1.10	1.11	1.07	1.10	1.10	1%
218	7	IM7/977	1.14	1.16	1.05	1.16	1.13	5%
195	10	IM7/MTM	1.28	1.39	1.12	1.33	1.28	9%
168	15	T8S/3900	1.79	1.89	1.60	1.82	1.78	7%



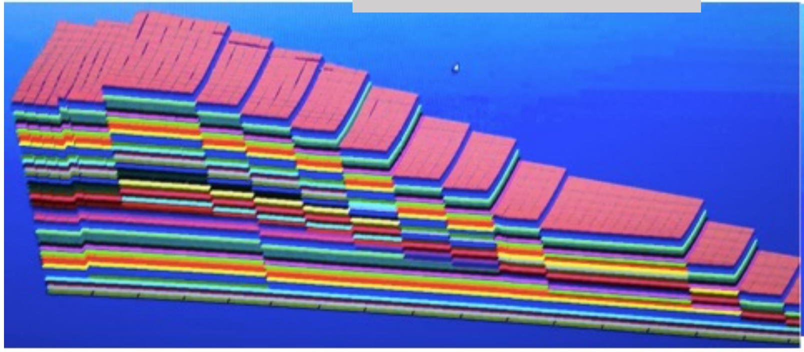


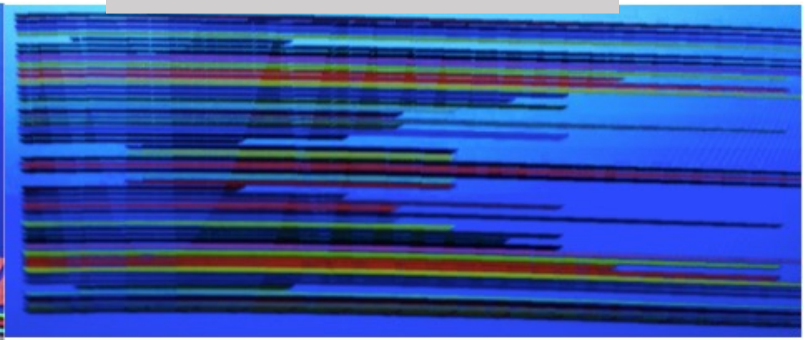
Better understood and more simplistic

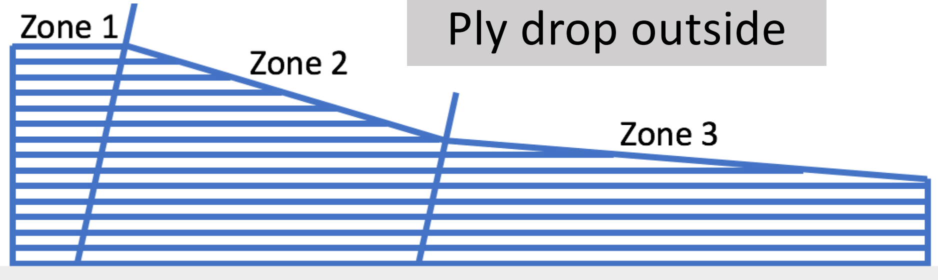
Keys to eliminate self-inflicted complexities

Aluminum	**The new CFRP**
Isotropy	Master stiffness: $A_{11} + A_{22} + 2A_{66}$ = Trace (one and only) Scalar product: $F_{ij}\sigma_i\sigma_j + F_i\sigma_i = 1$ (e.g., Tsai-Wu) Master failure criterion: omni envelopes (X and X’ only)
Homogeniety	Conditions: [A*] = [D*], [B*] = 0 (less complexity) Double-double: $[\pm\Phi/\pm\Psi]_{\mathbf{rT}}$ to replace $[0_p/\pm45_q/90_r]_{\mathbf{S}}$ Rating and scaling: instant answer without recalculation
Constant thickness	Additive manufacturing: tapered to save weight

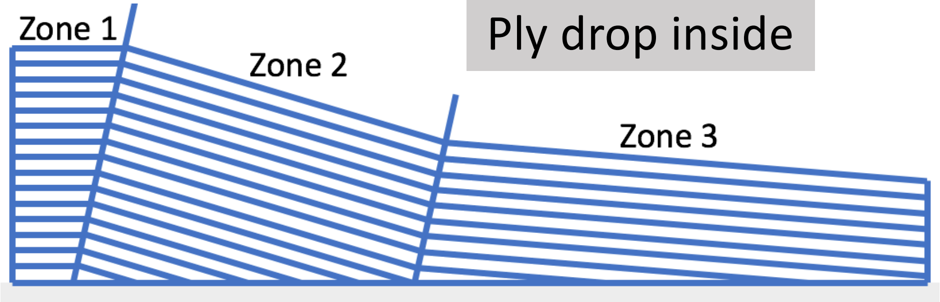
Legacy quad vs double-double: homogenization


Bricky stack


Internal discontinuities



Ply drop outside



Ply drop inside

Optimum: in- and out-of-plane homogenization where zones and taper transcend bays

Weight savings from tapered DD

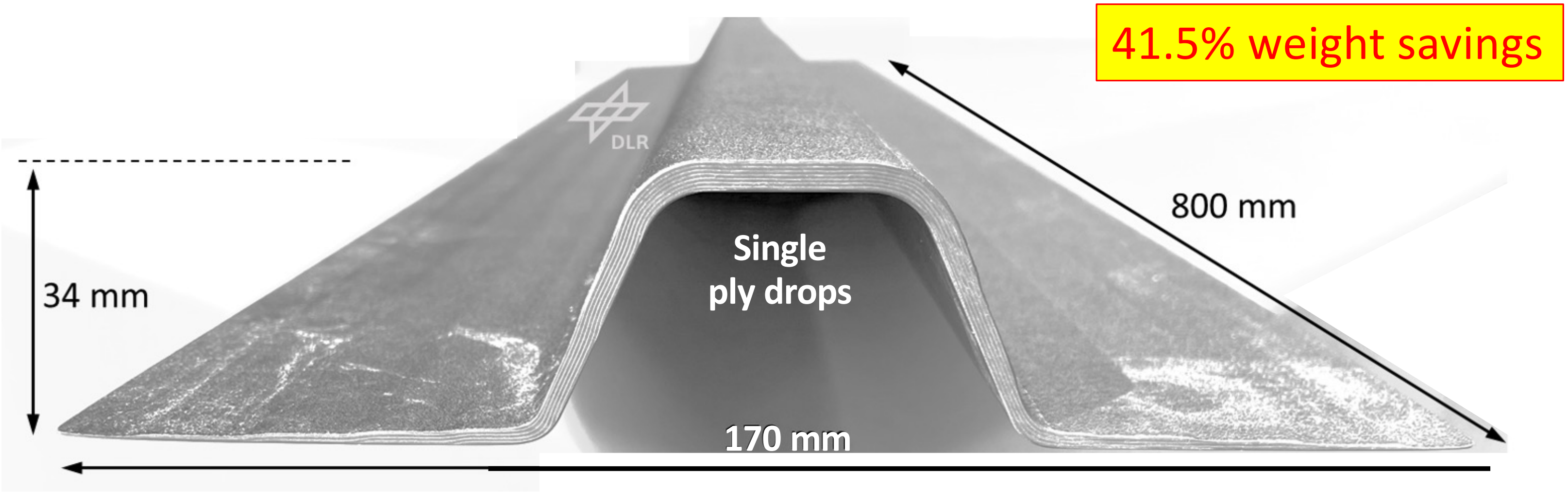
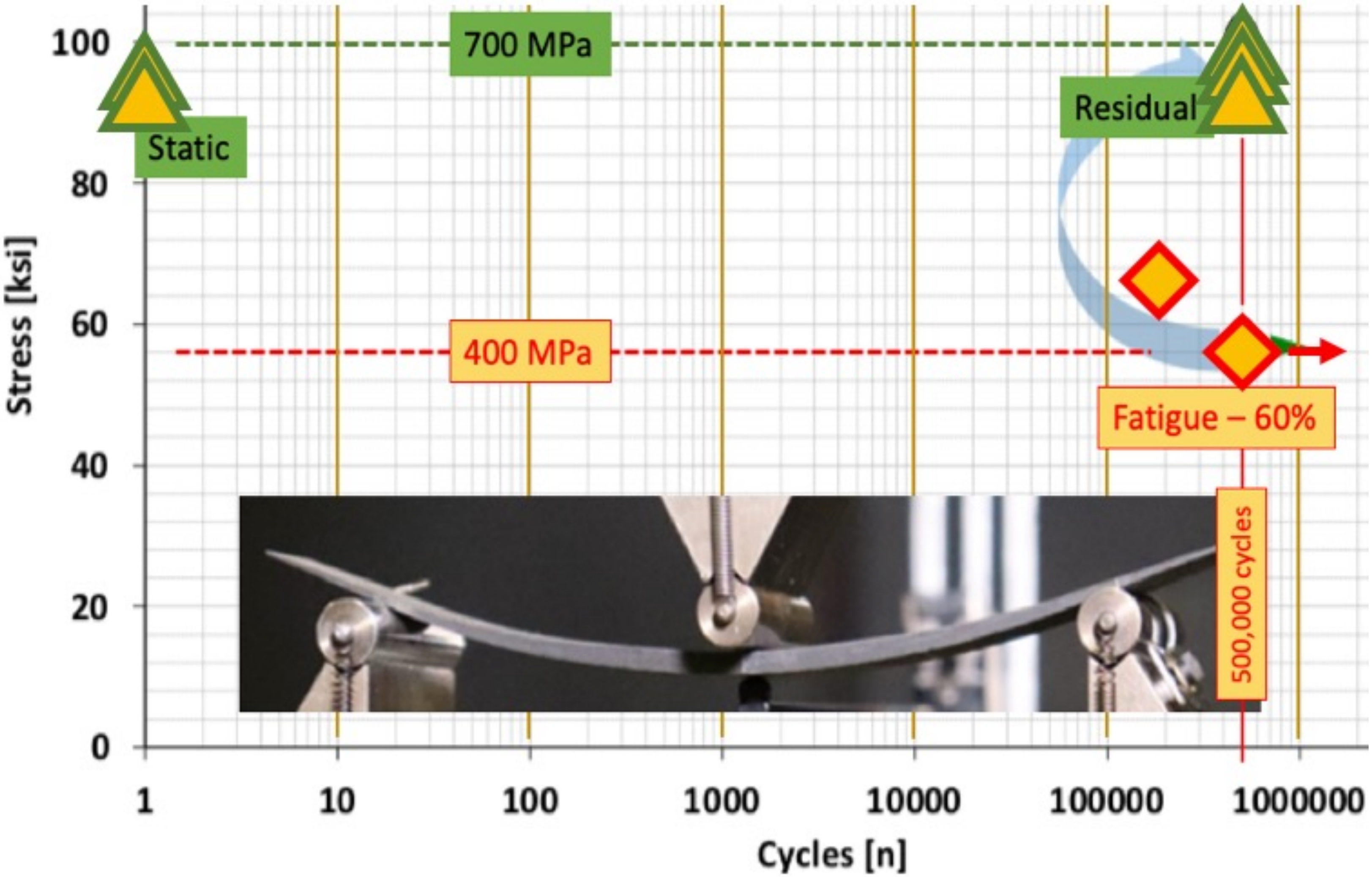
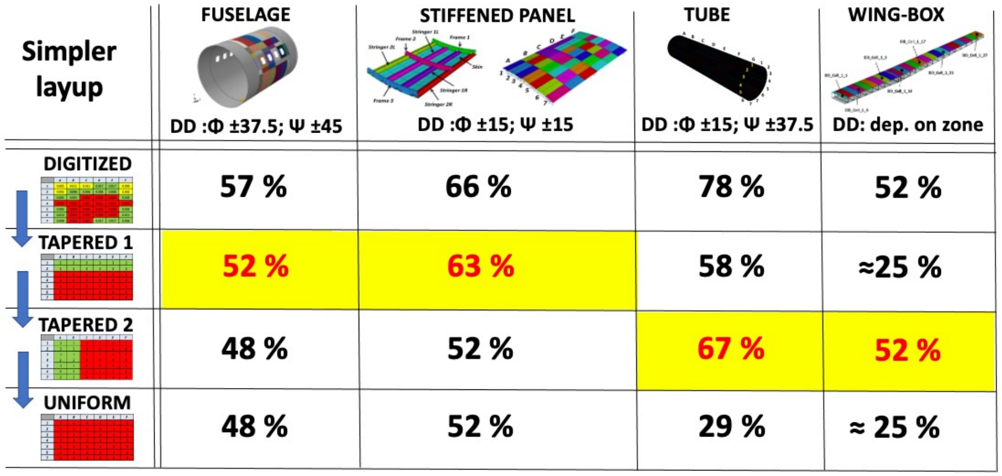


Figure 1 [$\pm 19.3/\pm 67]_{rT}$ with r = 1, ... 8, using card sliding technique, AS4/8552, single-sided diaphragm forming, autoclave cured at 180°C

Tapered [±0/±50] beam under fatigue

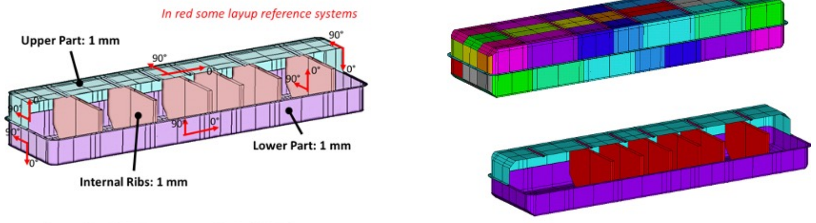


Weight savings in applications



Simpler layup	FUSELAGE DD :Φ ±37.5; Ψ ±45	STIFFENED PANEL DD :Φ ±15; Ψ ±15	TUBE DD :Φ ±15; Ψ ±37.5	WING-BOX DD: dep. on zone
DIGITIZED	57 %	66 %	78 %	52 %
TAPERED 1	52 %	63 %	58 %	≈25 %
TAPERED 2	48 %	52 %	67 %	52 %
UNIFORM	48 %	52 %	29 %	≈ 25 %

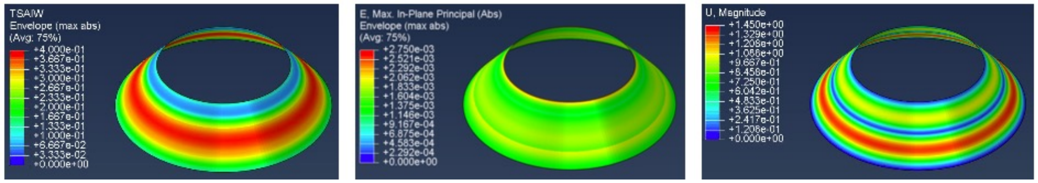
Airbus COMPOSITE FUEL TANK



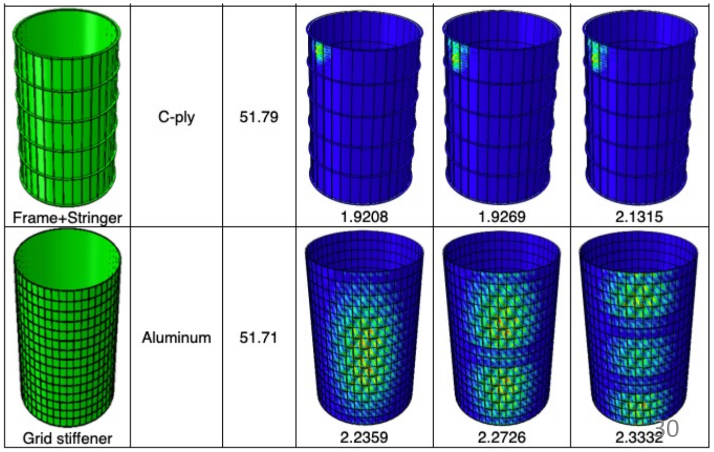
OPTIMIZED DD ANGLES

- basic NCF ply angle [30,-30] (Φ=30)
- Ψ angle = 0 in DD2 configuration ($[\pm\Phi]_{\pm\Psi}$)
- Internal frame layup: $[45,-45]_4$

38-40% WEIGHT SAVING

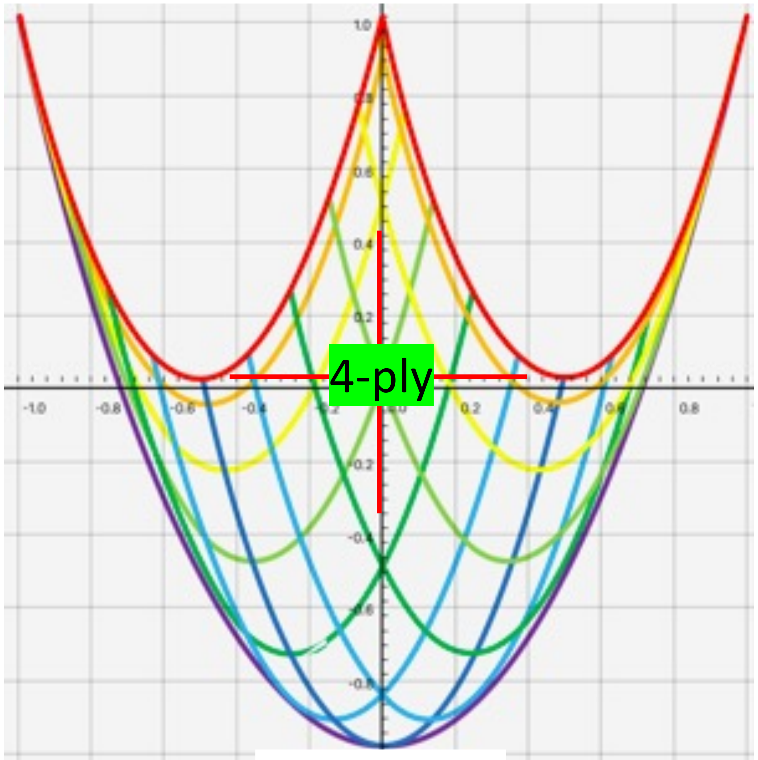


Description	Iteration	Critical R-value	Critical MS (strength)	Eigenvalue, 1st buckling mode	MS (buckling)	Natural Frequency (Hz)	Estimated Mass (for adapter shell)
Baseline QI	v41a	1.48	0.06	4.84	0.73	5.93	1007 lb
Optimized QI	v42c	1.48	0.06	3.17	0.13	5.59	783 lb *(-22%)*
Baseline [±25]	v50f	2.35	0.68	4.00	0.43	5.35	1007 lb *(-0%)*
Optimized [±25]	v51d	1.74	0.24	2.83	0.01	5.18	731 lb *(-27%)*



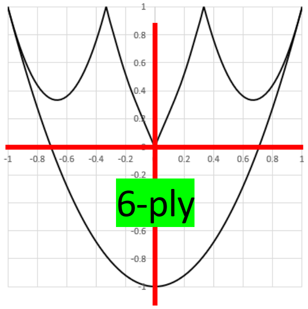
Frame+Stringer	C-ply	51.79	1.9208	1.9269	2.1315
Grid stiffener	Aluminum	51.71	2.2359	2.2726	2.3332

Field-base laminates in lamination-parameter plots

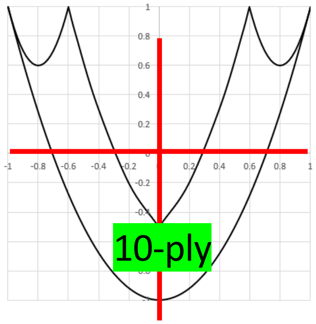


$[\pm\Phi/\pm\Psi]$

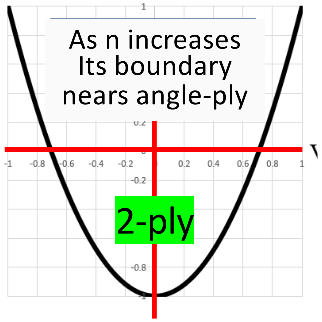
Double-double



$[\pm\Phi_2/\pm\Psi]$

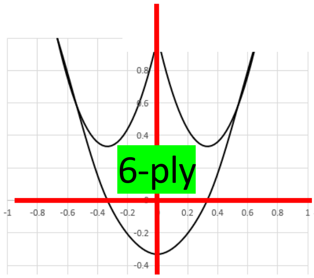


$[\pm\Phi_4/\pm\Psi]$

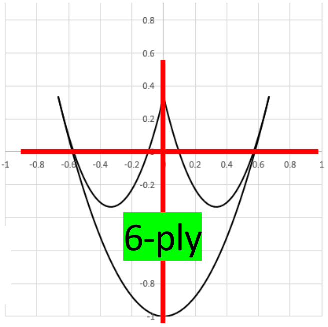


As n increases
Its boundary
nears angle-ply

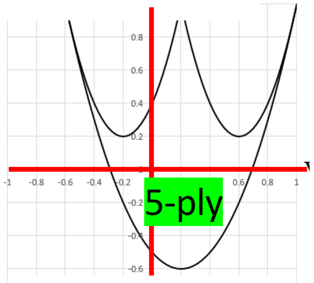
$[\pm\Phi_n/\pm\Psi]$



$[0/90/\pm\Phi/\pm\Psi]$

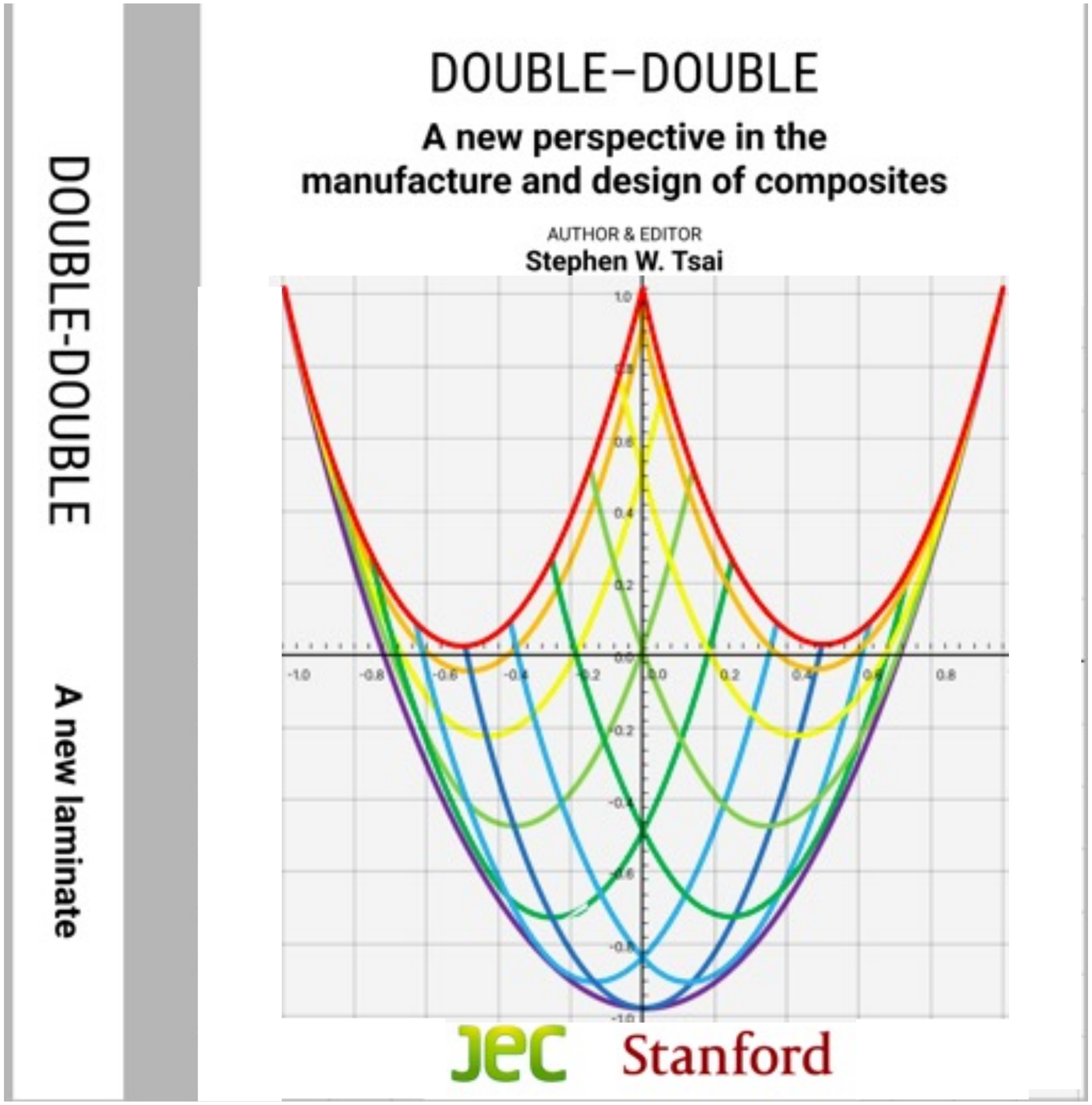


$[\pm45/\pm\Phi/\pm\Psi]$



$[0/\pm\Phi/\pm\Psi]$

Difficult to homogenize



DOUBLE-DOUBLE

A new laminate

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A new perspective in the manufacture and design of composites

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Better understood and more simplistic

Keys to eliminate self-inflicted complexities

Aluminum	**The new CFRP**
Isotropy	Master stiffness: $A_{11} + A_{22} + 2A_{66}$ = Trace (one and only) Scalar product: $F_{ij}\sigma_i\sigma_j + F_i\sigma_i = 1$ (e.g., Tsai-Wu) Master failure criterion: omni envelopes (X and X' only)
Homogeniety	Conditions: [A*] = [D*], [B*] = 0 (less complexity) Double-double: $[\pm\Phi/\pm\Psi]_{rT}$ to replace $[0_p/\pm45_q/90_r]_S$ Rating and scaling: instant answer without recalculation
Constant thickness	Additive manufacturing: tapered to save weight